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BENDING OF BEAMS ON AN ELASTIC SUBGRADE

A THESIS

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BENDING OF BEAMS ON AN

ELASTIC SUBGRADE

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Appreciation is due fellow graduate student L. R. Ritchie for his assistance in conducting the testing of the models.

To my wife, Dorothy, I express deep gratitude for her encouragement and assistance in the preparation of the manuscript.

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SUMMARY

The purpose of this investigation was to compare the conventional analysis for bending of beams on an elastic subgrade with experimental evidence. This objective was accomplished by determining the distribution of bending moment that occurred within steel beams of different rigidities acted upon by a variation of concentrated loads and resting upon a compacted, micaceous silt subgrade.

Load tests were conducted on three beams of different rigidities (but with identical widths and lengths) with loadings differing as a concentrated load in the center; equal concentrated loads at both ends; and three concentrated loads, one located in the center and two at the ends. SR-4 electrical strain gages, mounted on the beams, were used to determine the distribution of bending moment.

The results of the tests show that the error in computation of bending moments by the conventional analysis increases as the relative rigidity of the beam increases. Successive graphical differentiation of the bending moment curves shows experimental and computed contact pressures to be in considerable disagreement for relatively rigid beams. This disagreement manifested itself mostly as a concentration of pressure towards the ends of the beam, as the more rigorous analyses indicate.

The actual deflections followed closely the theoretical values at lower loads; however, at higher loads much greater deflections were

observed. This variation may be attributed to the decrease of modulus of elasticity at loads approaching failure, as indicated by triaxial and plate load tests.

The conventional analysis is satisfactory for finite beams in certain cases. For the cases of one concentrated load in the center and two concentrated loads, one at each end of the beam, the discrepancies are small for relatively flexible beams. For three concentrated loads, the bending moments were approximately the same for beams of different rigidities; however, the amount of discrepancy was considerable. The application of a correction factor to the conventional analysis for three loads is practical but needs more investigation.

Tests of this nature should be continued. An extension of tests to include other types of subgrade, effect of surcharge, and difference in beam widths is recommended. An accumulation of data of this type will be necessary before final conclusions can be made.

CHAPTER I

INTRODUCTION

The conventional analysis of beams on an elastic subgrade is based on the assumption that the ratio of contact pressure p to the deflection w is the same at every point of the beam. This assumption was first proposed by Winkler in 1867. It is often called Winkler's hypothesis and may be stated:

$$\frac{p}{w} = k = \text{constant}$$

where $k(\text{lb/in}^3)$ is a constant called coefficient of subgrade reaction.

For many years most of the investigation in this field has been of a theoretical nature, concerned with the solution of the basic differential equations involved. Slight attention has been given to the reliability of the basic assumption and it became customary to assume that k has a definite value for a given subgrade.

A thorough derivation of the conventional analysis has been presented by Hetényi (1). It is not the purpose of this work to present a full discussion of this entire theory but to question the reliability of the eventual analytical solutions. Therefore it will suffice to mention only that the solutions of the problem are obtained through a fourth order differential equation. This differential equation contains the damping factor λ which may be defined as:

$$\lambda = \sqrt[4]{\frac{K}{4 E_b I}} \tag{1}$$

where K = kB, and B = width of the beam; E_b = modulus of elasticity of the beam; and I = moment of inertia. In this form, λ includes the flexural rigidity of the beam with the factor K supposedly taking into consideration the deformation of the supporting medium. Since λ can influence the shape of the elastic line, its importance suggests that it be called the characteristic of the system with dimension of length^{-1}. Consequently λL or λx will be a dimensionless constant.

With λ thus defined, the coefficient K contained therein is the constant which has been under attack for many years. Hetényi (2) does not make any mention of the factors which determine its numerical value. Terzaghi (3) has published a paper dealing with the determining factors of k for sand and stiff clay under simple conditions and containing a discussion of errors introduced by simplifying assumptions.

Vesić (4) has shown that for beams of infinite length resting on an elastic-isotropic subgrade, the ratio of pressure to deflection along the beam is practically a constant defined by the expression:

$$K = kB = 0.65 \sqrt[12]{\frac{E_s B^4}{E_b I}} \frac{E_s}{1 - \nu_s^2} \tag{2}$$

This definition takes into account the nature of the subgrade by including the modulus of elasticity E_s, and Poisson's ratio ν_s of the subgrade. The flexural rigidity and width of the beam is also included.

Using this expression, he has indicated the procedure by which the value of coefficient k can be determined from laboratory or small scale field tests on the subgrade if the structural properties of the beam are known. This procedure for the determination of the coefficient was used throughout this work.

Theoretical analyses by DeBeer (5), Ohde (6), and Biot (7) showed that Winkler's hypothesis is generally not satisfied. However, their results implied that there still may be some cases where the conventional analysis could give fairly accurate results. These findings have been substantiated by recent theoretical and experimental investigations by Vesić (8). It is the purpose of this work to expand these investigations and provide additional experimental evidence.

The conventional analysis was chosen for comparison because of its relative simplicity for practical use in engineering problems. Large scale models were used to simulate the action of full scale foundations; the models are described in Chapter II together with the micaceous silt subgrade used as the supporting medium.

CHAPTER II

EQUIPMENT

The models tested and analyzed were basically three steel beams of varied cross-sections. With moments of inertia of these beams in the ratio 1:3.3:165, a wide range of flexural rigidities was investigated. The length of each beam was 72 inches and the width of each beam was eight inches.

The beams were loaded by concentrated loads applied by means of hydraulic jacks at each point of load application. Loads were measured by means of calibrated pressure gauges installed in the hydraulic systems. The load installations were devised so that free rotation in the longitudinal direction was assured. There were three loadings used for each beam: one concentrated load in the center; two concentrated loads, one at each end; three concentrated loads applied, one in the center and one at each end.

SR-4 electrical strain gages were applied by prescribed methods to the surface of each beam to measure strains due to bending. There were 26 strain gages on each beam and the symmetry of placement enabled each point under study to have six strain readings from which an average reading could be determined. The center position was an exception to this since two strain gages were used. A compensating gage was placed on each beam in such a way that it would not be affected by strains due to bending. Any temperature changes caused by atmospheric or subgrade conditions were accounted for by this compensating gage. A battery-operated Baldwin Type L strain indicator with switching and balancing unit was used to measure strains.

Deflections of the beam were recorded by twelve micrometer dial gages. These gages were placed symmetrically providing an average reading for each point under study.

Fig. 1 shows the complete test installation for the beam of medium flexural rigidity with three concentrated loads applied.

The 60 inch deep subgrade was compacted in two inch layers (after compaction). The test pit containing the subgrade was 12 feet long and eight feet wide. The subgrade consisted of a micaceous silt having the following characteristics (9):

plasticity index	8.0
% fines ($<$ 0.074 mm)	37
% clay ($<$ 0.002 mm)	3
void ratio	1.16
water content	26%
cohesion	9 psi
angle of friction, ϕ	23°

Deformation characteristics of the subgrade were determined by loading tests using an 18 inch circular and 24 inch square plate. Quick tri-axial tests have been made on four inch diameter samples extracted from the test pit. The test results are in the following table (10):

Table 1

Subgrade Deformation Characteristics

Test	Range of Pressure lb/in^2	$\frac{E_s}{1 - \nu_s^2}$ lb/in^2
Square plate 24 in.	0 - 28	1110
Circular plate 18 in.	0 - 25	1360
Triaxial 4 in. dia.	15 lb/in^2 (lateral)	1175
	30 lb/in^2 (lateral)	1200
	60 lb/in^2 (lateral)	1420

In computing deformation characteristics from triaxial tests, a value of Poisson's ratio of $\nu = 0.25$ was assumed.

CHAPTER III

PROCEDURE

The preparation of the subgrade and the test equipment followed the same pattern for each of the eight tests conducted. The rigid specifications followed for compaction of the subgrade and placing of all experimental equipment provided each test with idential initial conditions.

An equal amount of soil was used for each compacted layer. The same procedure of compacting with a Jay tamper was used for each layer. The high degree of compaction prevented any possible inconsistency in the compaction to have an effect in the quality of the subgrade. When the desired depth of subgrade was reached, the area where the beam was to be located was carefully leveled.

The loading columns were set exactly vertical. The points of load application were thoroughly cleaned and the load application devices placed so that the load would be transmitted as a true concentrated load.

All hydraulic connections were checked for leaks to assure true load readings. The two end loads were equal during the tests in which they were used and were connected to a common hydraulic pump by means of a high pressure T-connection. The center hydraulic jack was independent of the end jacks and was controlled by a separate hydraulic pump.

All micrometer dial gages were carefully placed at their intended points. These points were thoroughly cleaned. Each dial gage was carefully and securely fastened to the frame holding the dial gages. Each test was preceded by a careful dial gage setting at zero load.

The strain gage reading equipment was allowed to warm up sufficiently to give consistent readings. Each connection of strain gage to switching unit was securely tightened to prevent any distortion. Compensating gages were used in each test with the compensating gages directly applied to each beam.

The actual test procedure followed a general pattern. Loads were applied in increments to a maximum load either controlled by the maximum stress being reached in the beam or by the failure of the subgrade. Strain readings were made on each strain gage at every load increment through the application of the maximum load.

After each test was completed, a depth equal to three times the width of the beam (24 inches) was excavated and recompacted prior to the next test. This was done to account for the change in the modulus of elasticity of the subgrade brought about by the loading of the beam.

Prior to a few tests, a thin layer of lime was sprinkled on the top of each compacted layer at a small area under the beam. After the tests were completed, this portion was preserved intact and the plastic deformation of the subgrade together with the uniformity of compaction was observed.

All these precautions and rigid procedure provided extreme accuracy and kept inherent error to a minimum. Any variation in temperature was accounted for by the compensating gages. Frequent checks were made on identically placed strain gages to assure their giving equal strains. Deflections were also compared in this manner. The subgrade showed thorough homogeneity when the affected layers were observed. No tests were conducted with the sun heating particular portions of the beams tested. The loading was maintained constant while strain readings were being made. During the interim between tests, care was taken not to allow any change of moisture in the subgrade.

Each beam was placed in a testing machine and a known bending moment applied for purposes of calibration. The calibration was conducted under the same conditions as existed during the testing. Calibration graphs were then plotted and the experimental strains compared with these graphs to provide an accurate value of bending moment. Upon reloading in the testing machine, strain readings were identical within five micro-inches per inch.

Fig. 1 shows an actual test in progress.

CHAPTER IV

DISCUSSION OF RESULTS

A. General Comments

Few model analyses of beams on an elastic subgrade have been made in the past; only tests by DeBeer (11) and Vesić (12) have been cited in literature. From this previous work there were indications that considerable variations between the actual values of bending moments and values obtained by the conventional analysis may exist for certain cases, while in other cases the conventional analysis may provide the accuracy necessary to satisfy most engineering problems. The primary intent of this study was to make comparisons between the conventional analysis and experimental evidence for various cases, extending the work conducted by Vesic.

To accomplish this, the distribution of bending moment in three beams of different flexural rigidities was found through the use of strain gages applied to each beam. The shear and contact pressure was then obtained by graphical differentiation of the bending moment curves. Actual deflections were obtained by using micrometer dial gages. The actual values of K were determined by computing the ratio between the contact pressures obtained and the actual deflections observed. All experimental values were compared with values obtained by the conventional analysis and both computed values and values obtained from test results are presented in graphical form in the appendix.

Using expressions (1) and (2), with $E_s/1 - \nu_s^2 = 1200$ lb/in^2, the following values of K and λL for the model beams were obtained:

Table 2

Beam Characteristics

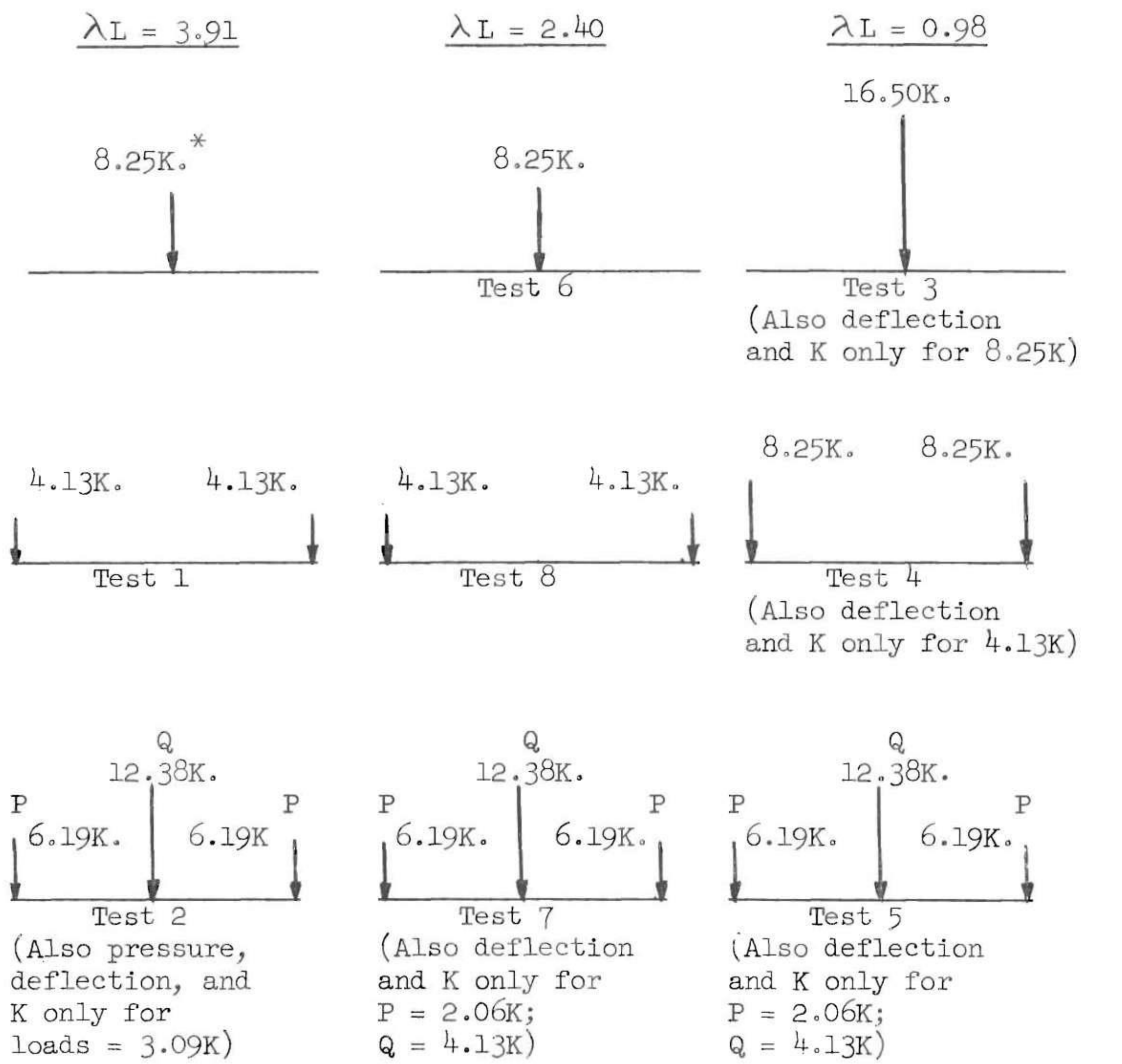
	Beam	I	K	λL
	8x1	0.667 in^4	694 lb/in^2	3.91
	SC8A	2.0 in^4	585 lb/in^2	2.40
	8 WF 31	109.7 in^4	454 lb/in^2	0.98

The values for all conventional analyses were determined by the use of formulas presented by Hetényi for one concentrated load in the center (13) and for a concentrated load two inches from each end (14). For the cases of three concentrated loads, the theoretical values were obtained by superposition of the end load and center load values.

The particular loadings analyzed and compared for each beam are listed in Table 3:

Table 3

Loadings Analyzed



*Analysis for this case from Vesić, A. B., "Beams on Elastic Subgrade and the Winkler's Hypothesis," Proceedings of the Fifth International Conference on Soil Mechanics and Foundation Engineering, Paris 1961. (in press)

B. Interpretation of Test Results

For $\lambda L = 3.91$

Load in center.--The results presented in Fig. 2 are those obtained by Vesić (15) and are reproduced here for purposes of comparison. The maximum bending moment obtained by test results was 10 per cent lower than the computed value. The deflections and pressures had approximately the same experimental and computed values. The K values had slight variation from the computed values of K.

Test 1; loads at each end.--The test results are shown in Fig. 3 and Fig. 4. The experimental bending moment in the center was 10 per cent higher than the computed value but the maximum values agreed. The variation in the center comes evidently from the fact that theoretical values were obtained assuming the possibility of negative pressure (see the pressure diagram). The diagram containing the K values indicates that K can have considerable variation, due to the low values of both pressures and deflections near the center of the beam. The values of K in this sector are questionable and were not plotted.

In conclusion, the bending moments, shears, contact pressures, and deflections were in fairly close agreement with computed values for this test.

Test 2; three loads.--The test results are shown in Fig. 5 and Fig. 6. The maximum negative bending moment was observed to be 32 per cent lower than computed values, whereas the maximum positive bending moment was observed to be 32 per cent higher than the computed maximum positive bending moment. The shears and contact pressures agreed fairly well; however the deflection and K values were in disagreement at the higher loads. This can be explained by the decrease of the modulus of elasticity E_s when approaching failure, the secant modulus at higher loads in both triaxial and plate load tests being considerably lower than the initial tangent modulus E_s which was used throughout the computations (see also Figs. 19 and 20). The contact pressure, deflection, and K values for lower loads, as indicated in Fig. 6, showed closer agreement with K still varying slightly from the computed value.

For $\lambda L = 2.40$

Test 6; load in center.--The test results are shown in Fig. 7 and Fig. 8. The maximum bending moment obtained was in close agreement with the computed maximum bending moment, but a difference in the bending moment values were observed in the vicinity of the quarter points of the beam. The higher experimental bending moments at the quarter points can be explained by a concentration of pressure at the beam ends which also affected K values. The deflections were affected little by this difference in bending moment at the quarter points.

Test 8; loads at each end.--The test results are shown in Fig. 9 and Fig. 10. A 4 per cent lower maximum bending moment was observed here. The concentration of contact pressure at the beam ends evidently lowered the maximum bending moment. The deflections were in fairly close agreement although the K values showed considerable variation along the beam.

Test 7; three loads.--The test results are shown in Fig. 11 and Fig. 12. Maximum negative bending moments were 32 per cent lower with 100 per cent higher maximum positive bending moments observed. The contact pressure showed considerable variation as did the deflection at the high loads being analyzed. With a lower load analyzed, the effect was the same as occurred for the similar case of loading with $\lambda L = 3.91$.

For $\lambda L = 0.98$

Test 3; load in center.--The test results are shown in Fig. 13 and Fig. 14. Somewhat higher (16 per cent) maximum bending moment was obtained than the maximum bending moment computed for this characteristic length. The difference in the contact pressure distribution became more apparent now. The deflection diagram in Fig. 14 shows variation due to a higher load analyzed and with a load equal to the loads analyzed for $\lambda L = 3.91$ and 2.40, the computed deflections and actual deflections are in close agreement. This also can be attributed to the change in modulus of elasticity of the subgrade becoming effective at higher loads.

Tests 4; loads at each end.--The test results are shown in Fig. 15 and Fig. 16. The maximum bending moment was 14 per cent lower in this case than the computed bending moment value. The deflection and K phenomena occurring with the load in the center occurred here also, and the analysis with a lower load is indicated in Fig. 16.

Test 5; three loads.--The test results are shown in Fig. 17 and Fig. 18. The load analyzed here was identical with the load analyzed for $\lambda L = 3.91$

and $\lambda L = 2.40$. The maximum negative bending moment was 22 per cent lower and the maximum positive bending moment 200 per cent higher than computed values. The contact pressure indicates more variation than in the other two beams also. However the values of K can be brought to closer agreement with the analysis of a lower load. This is shown in Fig. 18.

C. Further Comments on Test Results

Intermittent cracks in the subgrade around the periphery of the beam were observed when the total loading on the beams reached approximately 12 kips. Considering both internal friction and cohesion and using Terzaghi's bearing capacity factors, ultimate bearing capacity of 110 kips for general shear failure and 43 kips for local shear failure was computed.

However, it was observed in all the tests that the maximum pressure, occurring for a total loading at which failure of the subgrade began, was close to the computed bearing capacity of the subgrade considering the subgrade to be frictionless with $q = 5.7c$ as Terzaghi proposes (neglecting friction).

Vesić (16) has presented a limiting value of $\lambda L > 2.25$, when used for a finite beam acted upon by a concentrated load, to give reasonable accuracy using the conventional analysis with a constant k. He has found also that when loads are acting at the ends reliable estimates of bending moments can be made by the superposition of M' due to the middle loads and 0.83 M" where M" is due to the end loads. In the case of three loads,

$$M = M' + 0.83\, M'' \qquad (3)$$

The results of this correction to the computed values are shown in Fig. 11 and Fig. 17. Investigation of this correction indicates that the 0.83 factor for end load moments may be a variable depending on λL. Further investigation should be conducted concerning correction factors before general deductions can be made.

Fig. 22 shows the difference between the actual bending moments and computed bending moments to increase with the increased flexural rigidity of the beam. The limit of $\lambda L > 2.25$ for accuracy in the conventional analysis of bending moments is observed in Fig. 22 to be reasonably reliable.

Fig. 23 shows that there is little difference in the observed maximum bending moments due to three loads, for any range of flexural rigidity. However it does indicate that significant difference exists between the computed values and the values obtained from test results.

Accuracy of Results.--The definite trend of the test results indicates good accuracy was achieved. The inherent error in the use of strain gages was at a minimum. The accuracy of the dial gages was assured when each dial gage used in each test did not indicate any error when plotted on load vs. deflection calibration graphs.

The compaction of the subgrade showed good homogeneity when intact areas were checked to a depth of two feet. Symmetrical deflection under symmetrical loading also indicated good homogeneity was achieved in the subgrade.

In general, the accuracy of the results was better than might be expected in tests of this nature.

CHAPTER V

CONCLUSIONS

The following conclusions can be made from the results of these tests. It should be kept in mind that they apply only to the performance of finite beams resting on a slightly cohesive micaceous sandy silt subgrade.

1. The amount of error in the conventional analysis depends on the rigidity of the beam. It seems that the criterion $\lambda L > 2.25$ is showing reasonably well the limit beyond which conventional analysis still may be reliable.

2. For one concentrated load in the center of beams, the actual maximum bending moments can be as much as 16 per cent higher (for relatively rigid beams) or three per cent lower (for flexible beams), when compared with the conventional analysis.

3. For two concentrated loads, applied at the ends of beams of different rigidity, the actual maximum bending moments have a difference extending from 14 per cent lower for relatively rigid beams to exact agreement for flexible beams, when compared with the conventional analysis.

4. For three concentrated loads, one load acting in the center and two at the ends of the beam, the actual maximum bending moments seem not to vary much with the rigidity in the beam. However when comparison is made with the conventional analysis, the difference in maximum positive bending moments extends from 200 per cent higher for relatively rigid

beams to 32 per cent higher for flexible beams while the difference in maximum negative bending moments extends from 22 per cent lower for relatively rigid beams to 32 per cent lower for flexible beams when compared with the conventional analysis.

This discrepancy between the test results and the results obtained by the conventional analysis comes from the concentration of the contact pressures at the ends of the beams. The more rigorous analyses take into account the concentration of pressure and show closer agreement with the results obtained from tests.

5. The correction factor 0.83 for moments due to the end loads found by the conventional analysis provides fairly satisfactory results.

6. The results of these tests verify Vesić's expression for the value of K as well as the procedure for its determination by laboratory and small scale field tests.

7. The observed deflections agree closely at lower loads, however disagreement occurs at higher loads due to the change in the modulus of elasticity of the subgrade.

CHAPTER VI

RECOMMENDATIONS

In order to make final conclusions regarding the bending of beams on a subgrade, it is recommended that experimental studies be continued. The studies should be extended to include other types of subgrades like sand and clay. The effect of surcharge could be included in tests for each soil type used as subgrade.

Additional cases of loading should be included, such as unsymmetrical and uniform loads. The amount of load applied should be increased until failure of the subgrade is reached and a full analysis of bending moment, shear, contact pressure, and deflection made to determine the stress distribution at subgrade failure.

The positioning of the strain gages should be analyzed to give more points under consideration. This would provide a more accurate bending moment curve. Strain gages should be placed at points where maximum values can be anticipated and also near the ends to provide a more accurate indication of end conditions. The strain gages used in the beams of greater flexural rigidity should have a higher sensitivity to give large enough strain indications for the lower stresses obtained in more rigid beams.

Further investigations may be conducted concerning a correction factor applicable to the conventional analysis. A correction factor for M' values and variation of correction factors depending on λL should also be considered before any definite conclusions can be made concerning the usefulness of such an approach.

Results of experimental investigations should also be compared with the results of more rigorous analyses for beams on elastic foundations, like those by DeBeer and Ohde.

APPENDIX

NOTATION

b = half-width of the beam (in.)

k = coefficient of subgrade reaction (lb/in^3)

p = pressure at the contact between the beam and the subgrade (lb/in)

w = deflection of the beam (in)

x = abscissa along the beam (in)

B = width of beam = 2b (in)

E_b = Young's modulus for the beam (lb/in^2)

E_s = Young's modulus for the subgrade (lb/in^2)

I = moment of inertia of the beam (in^4)

K = modulus of subgrade reaction defined by (2) = kB (lb/in^2)

L = length of the beam (in)

M = bending moment in the beam (kip-in)

M' = computed bending moment for concentrated loads in middle of beam (kip-in)

M'' = computed bending moment for concentrated loads acting at the ends of the beam (kip-in)

P = a concentrated load (kip)

Q = a concentrated load (kip)

V = shearing force in the beam (kip)

λ = damping factor expression defined by (1) (in^{-1})

ν = Poisson's ratio for subgrade

q = bearing capacity (lb/in^2)

c = cohesion (lb/in^2)

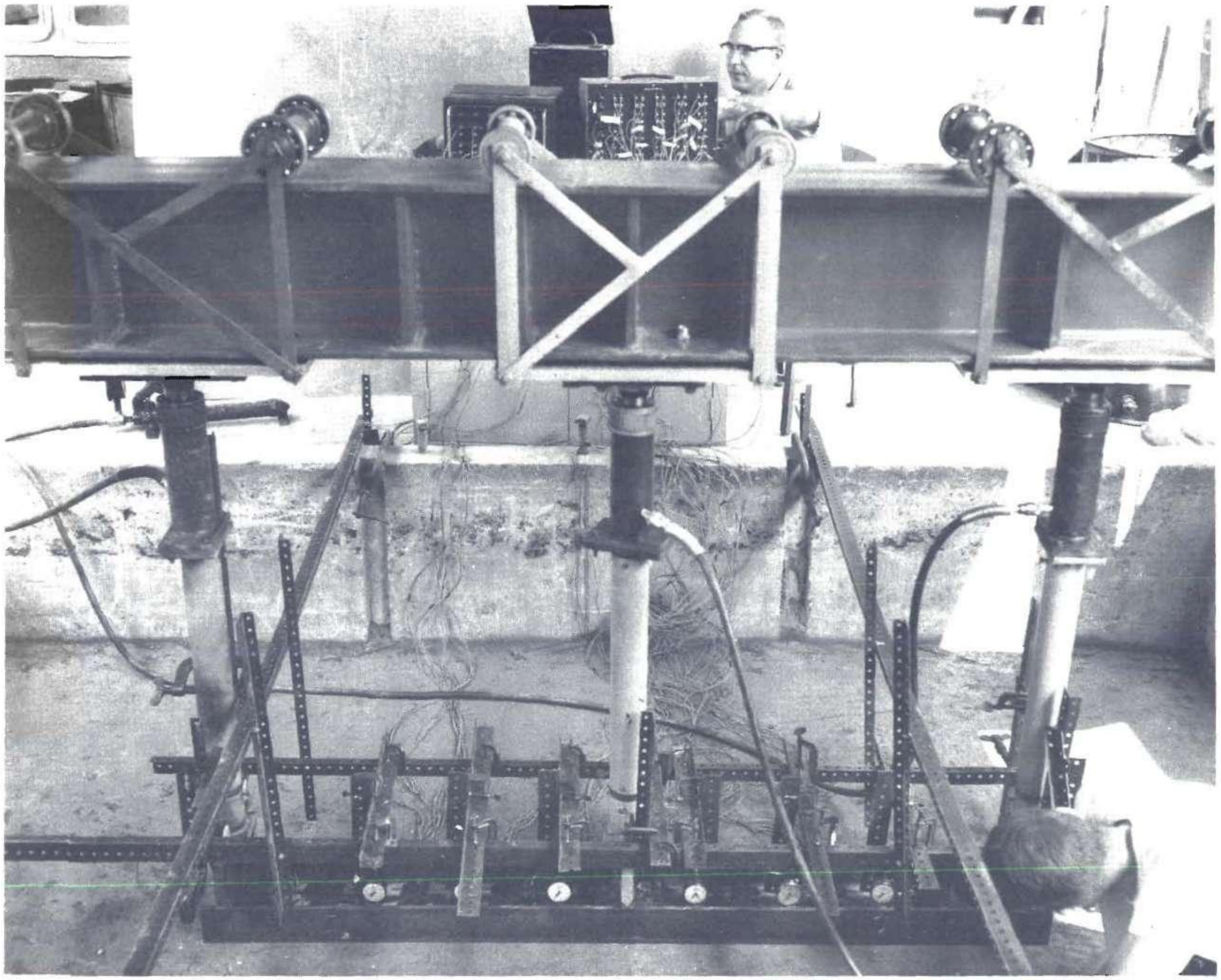


Figure 1. Test Installation.

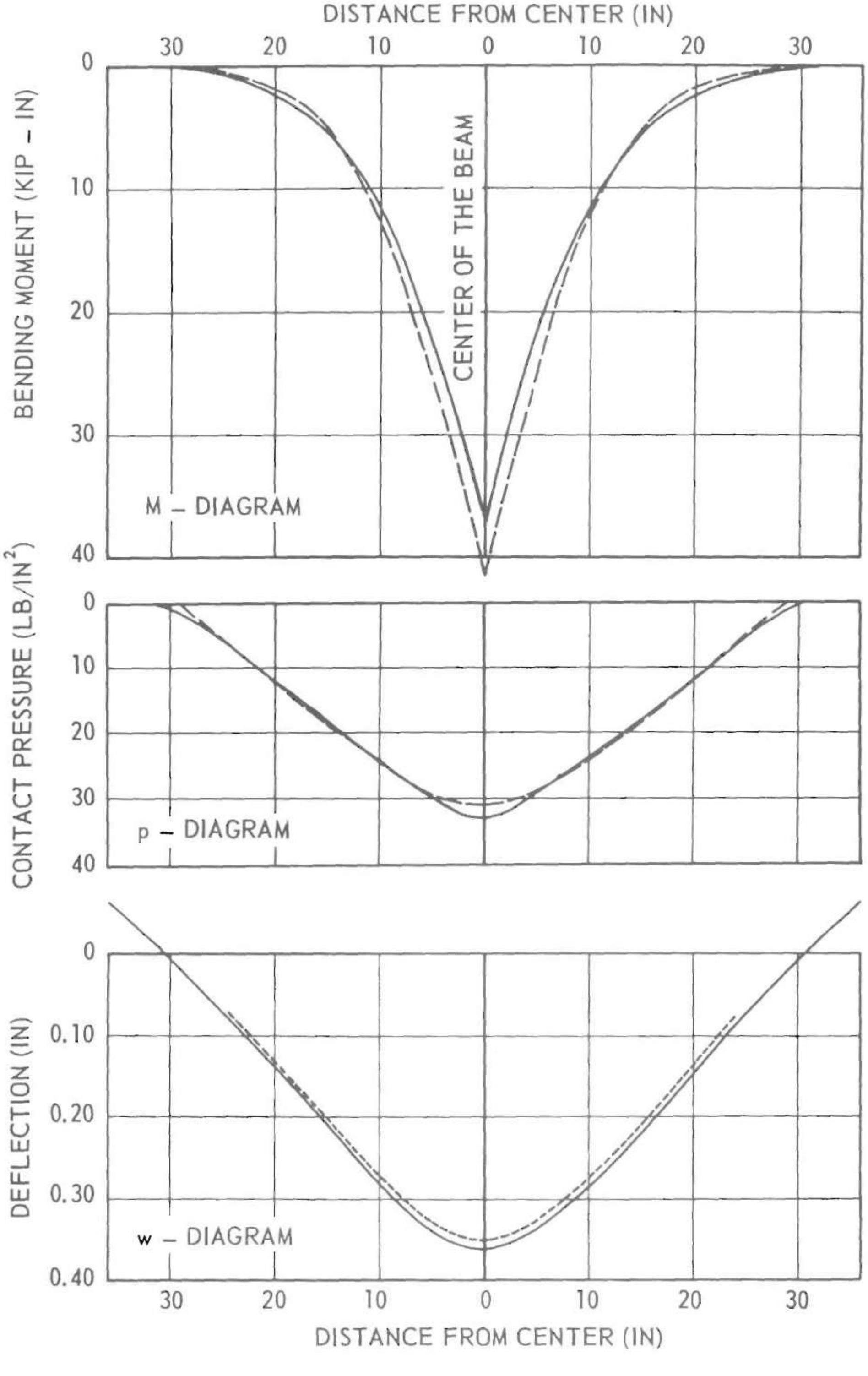


Figure 2. Values of Bending Moment, Contact Pressures, and Deflections for $\lambda L = 3.91$ and Concentrated Load of 8.25 K. in Center as Obtained from Test Results (Full Lines). Dotted lines show the corresponding computed values.

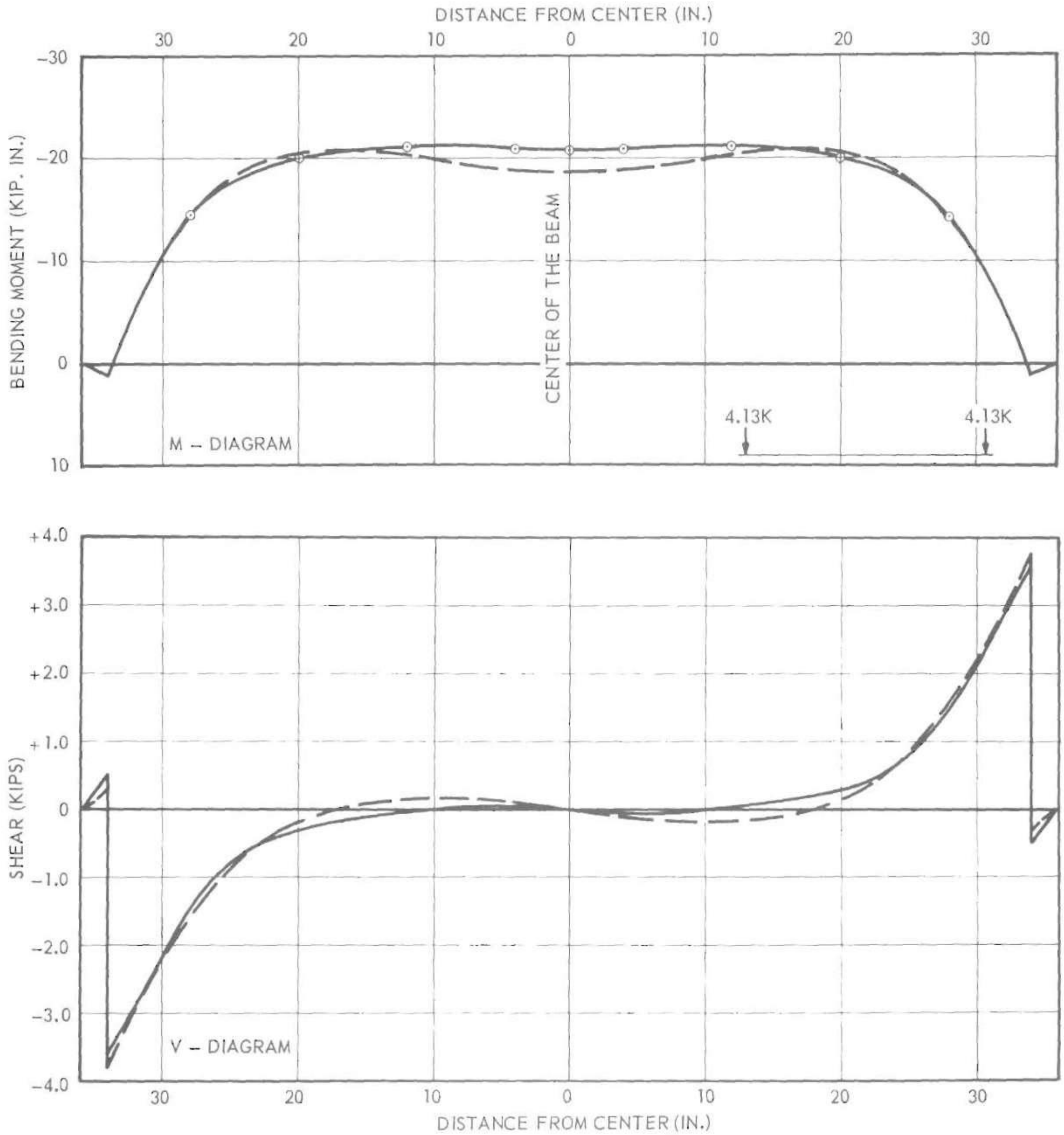


Figure 3. Values of Bending Moment and Shear for $\lambda L = 3.91$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

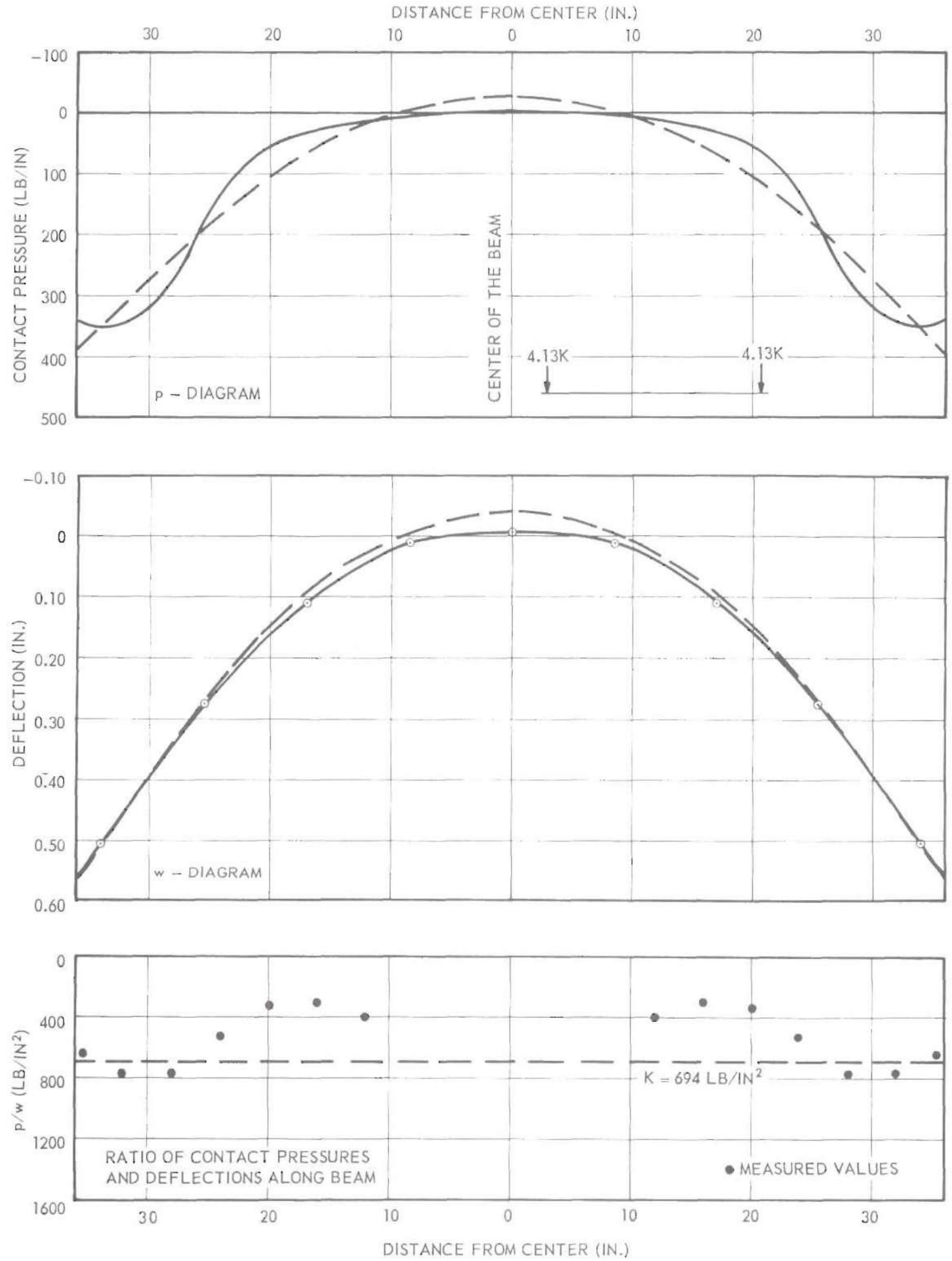


Figure 4. Values of Contact Pressures, Deflections, and K for $\lambda L = 3.91$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

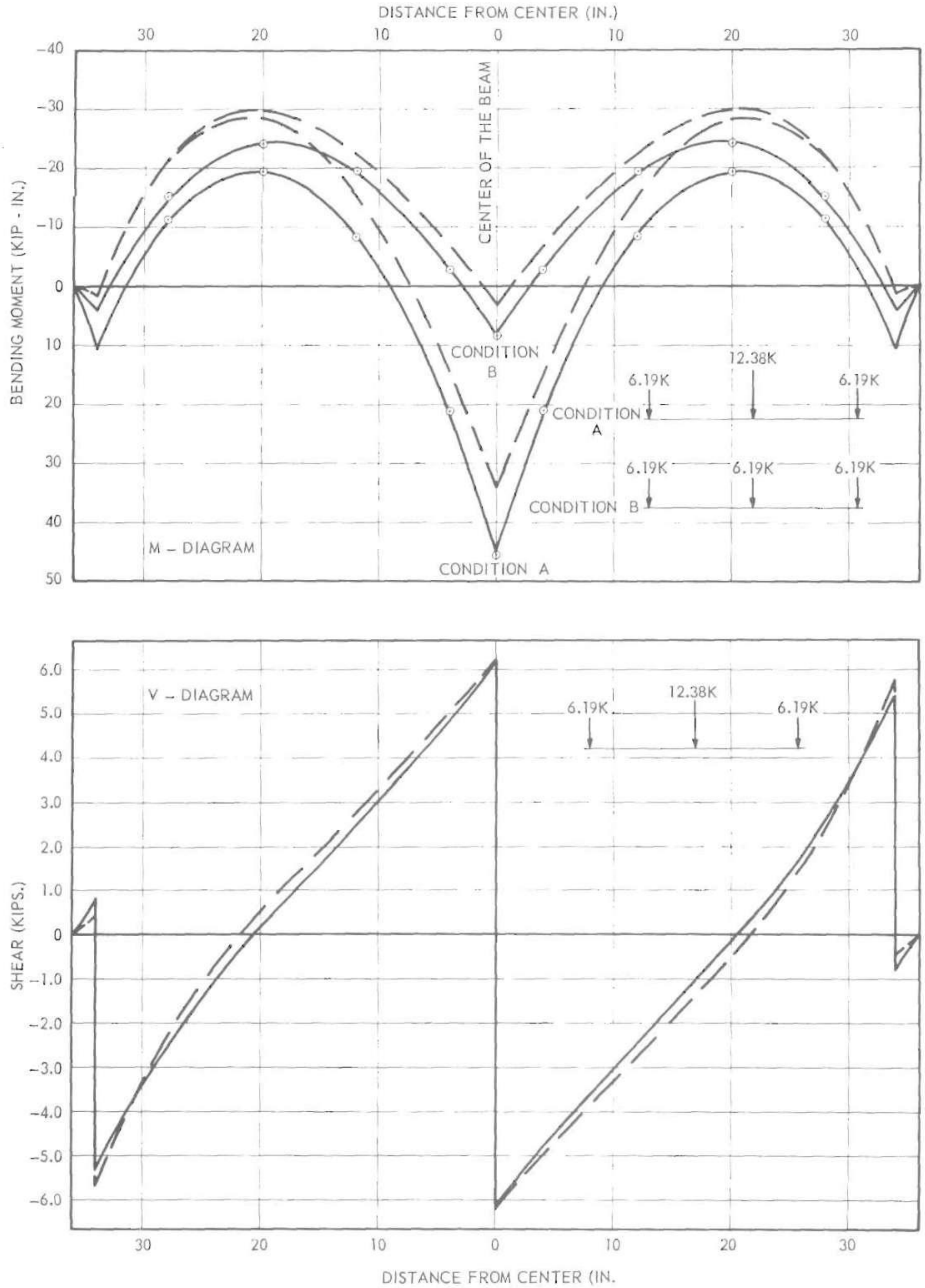


Figure 5. Values of Bending Moment and Shear for $\lambda L = 3.91$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

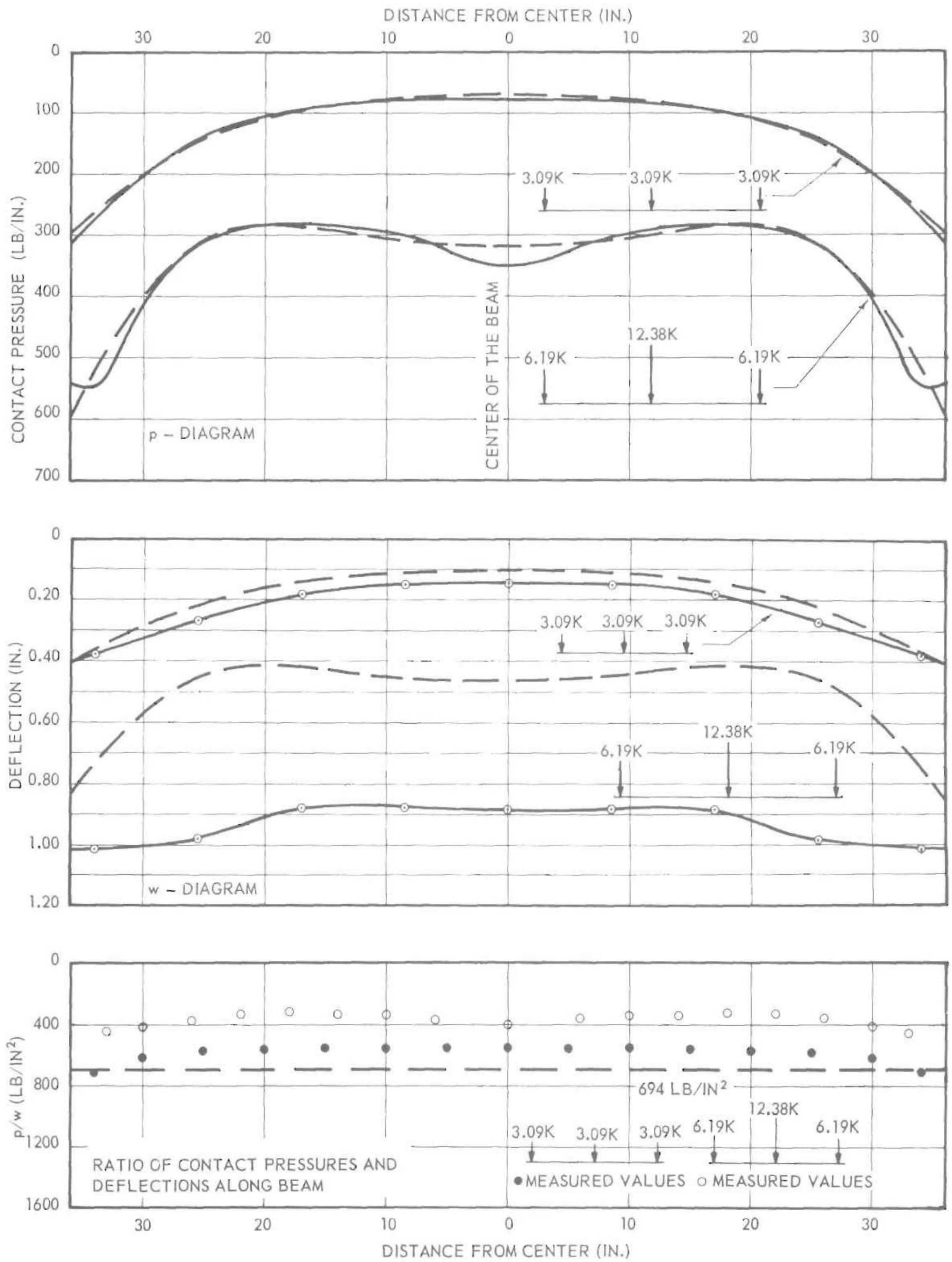


Figure 6. Values of Contact Pressures, Deflections, and K for $\lambda L = 3.91$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

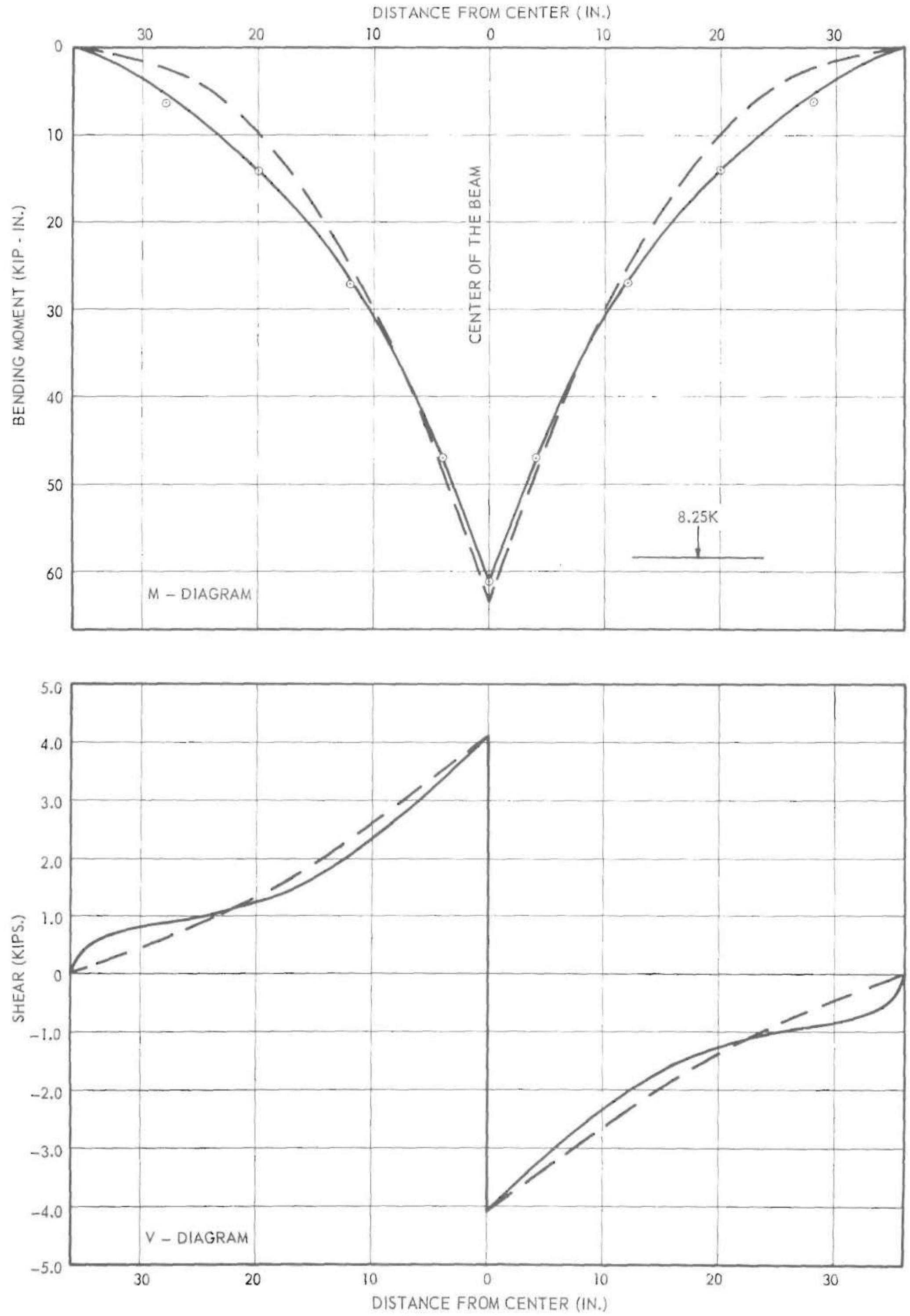


Figure 7. Values of Bending Moment and Shear for $\lambda L = 2.40$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

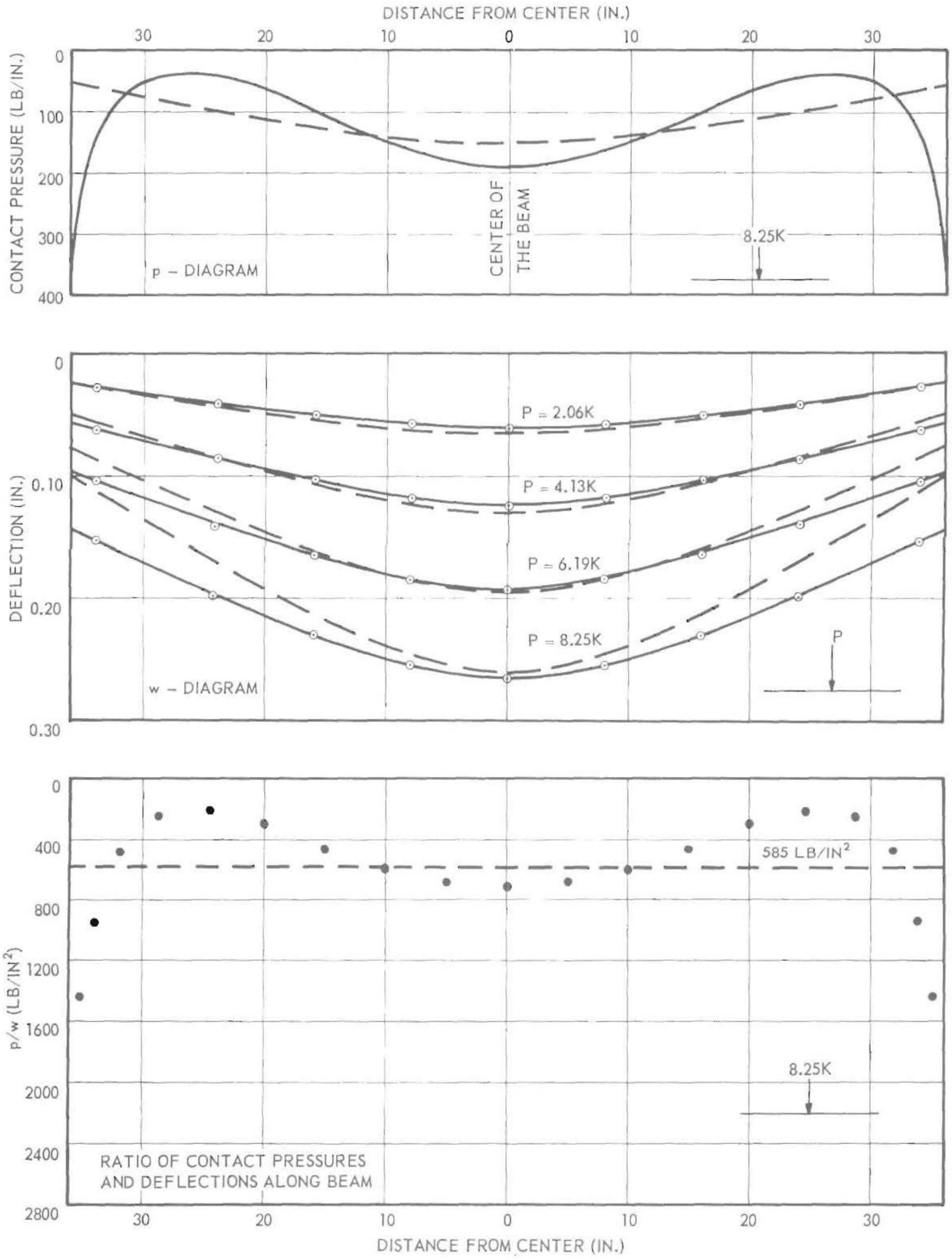


Figure 8. Values of Contact Pressures, Deflections, and K for $\lambda L = 2.40$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

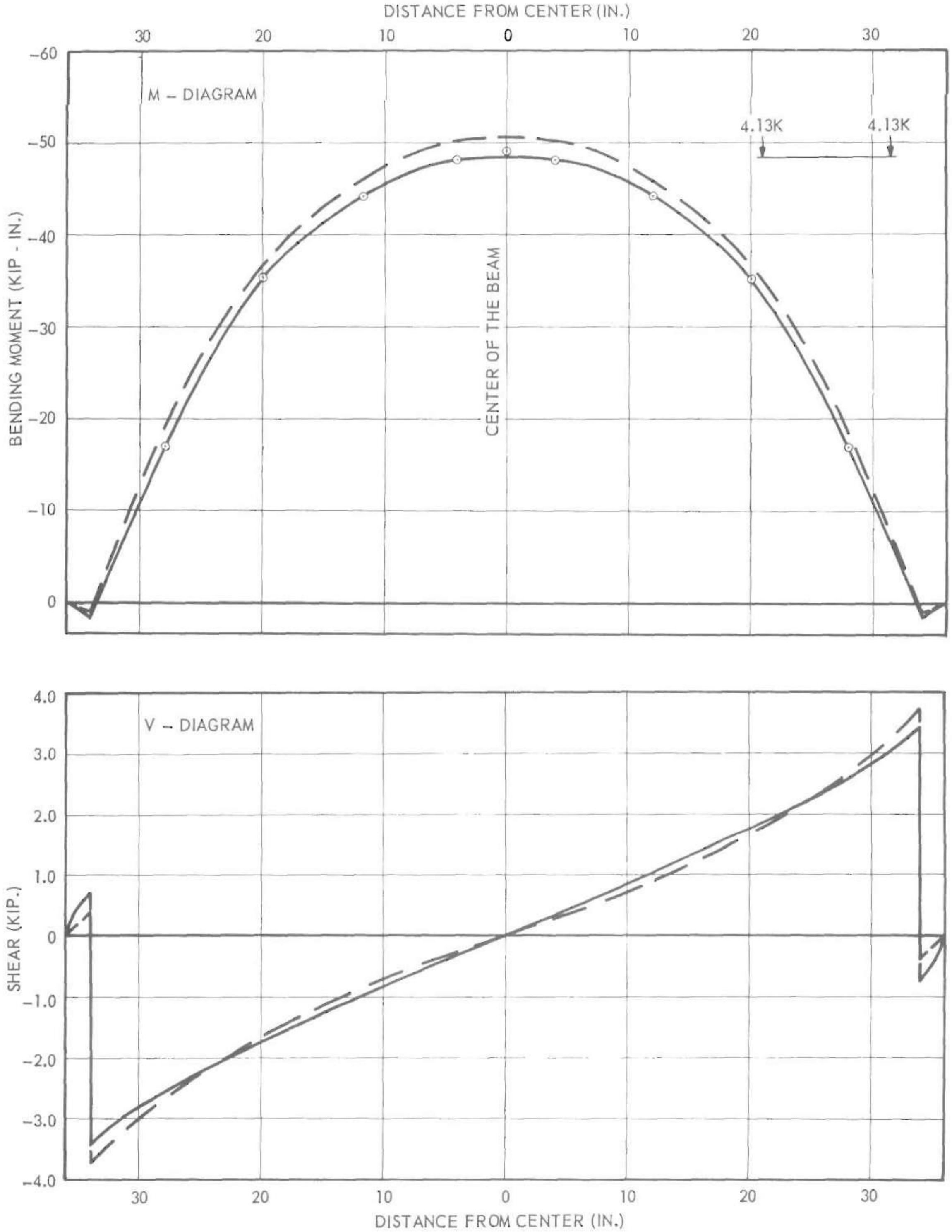


Figure 9. Values of Bending Moment and Shear for $\lambda L = 2.40$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

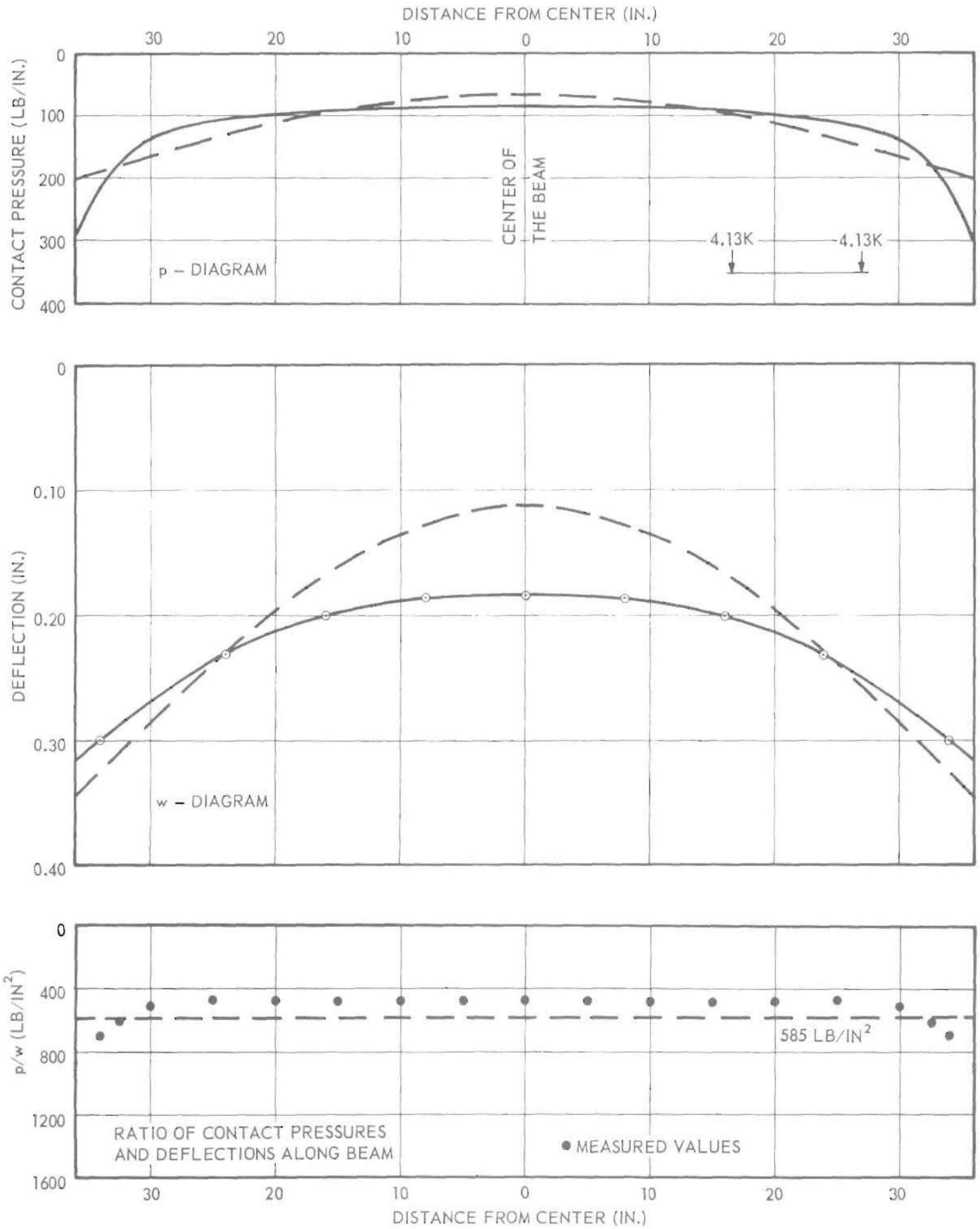


Figure 10. Values of Contact Pressures, Deflections, and K for $\lambda L = 2.40$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

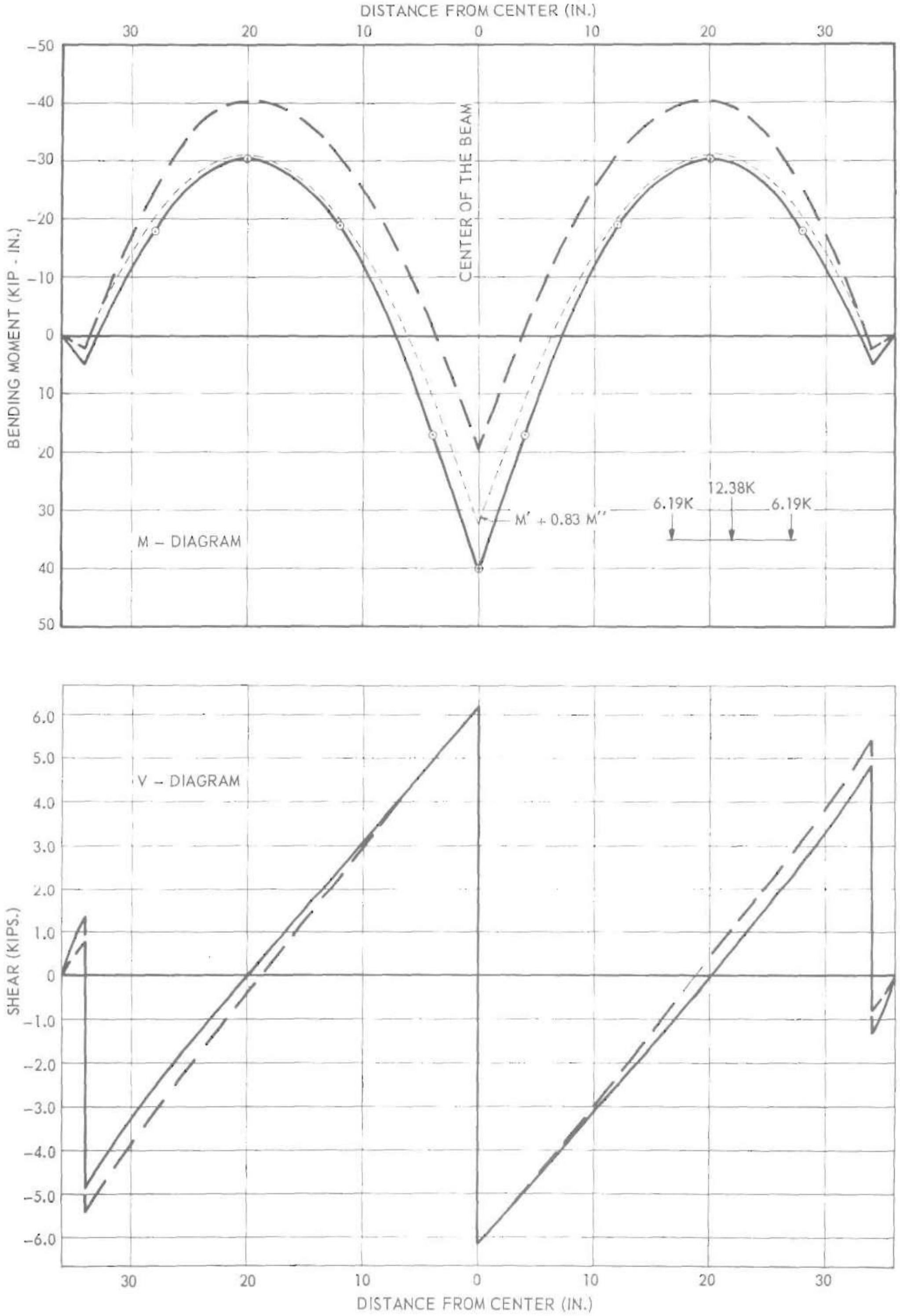


Figure 11. Values of Bending Moment and Shear for $\lambda L = 2.40$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

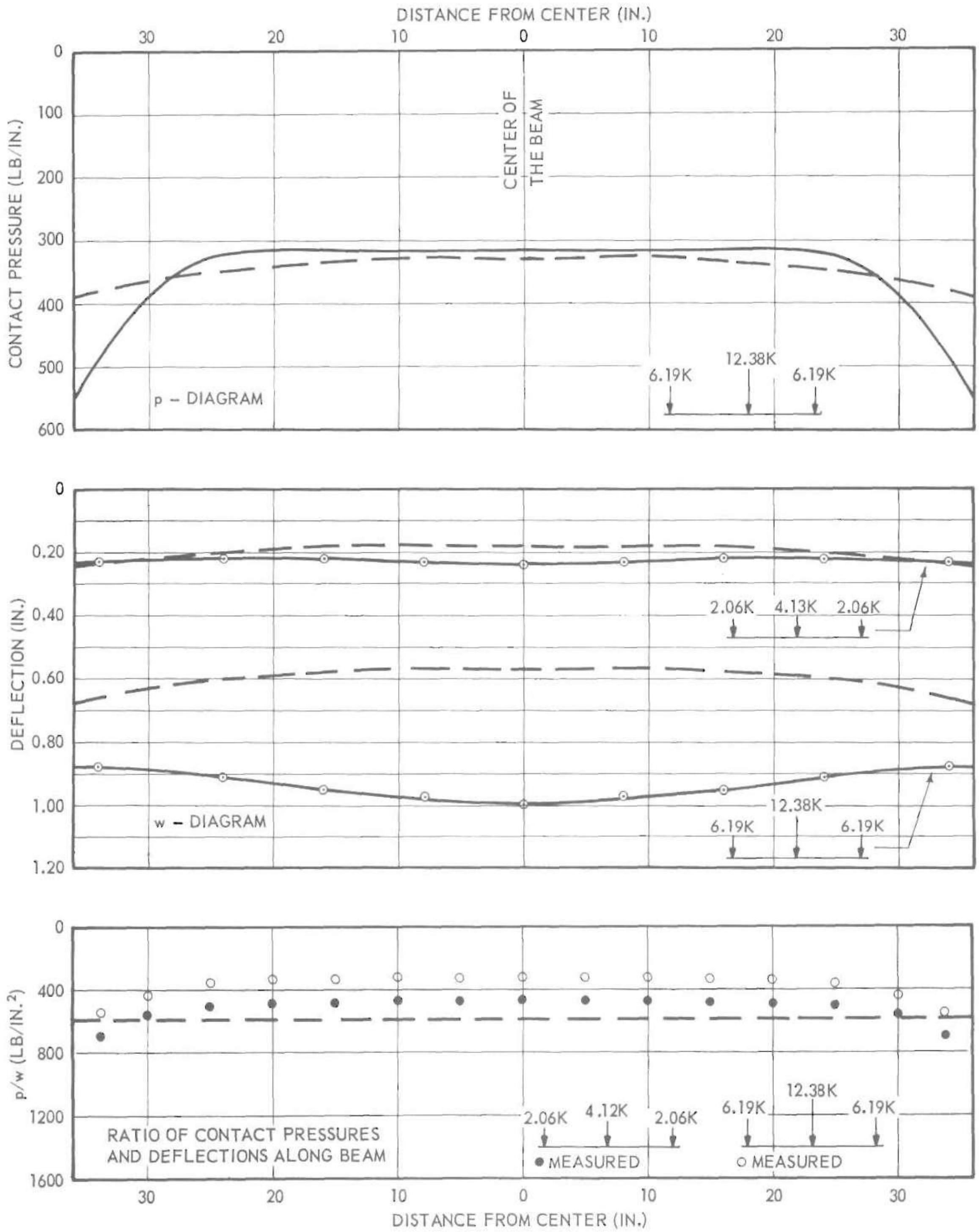


Figure 12. Values of Contact Pressures, Deflections, and K for $\lambda L = 2.40$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

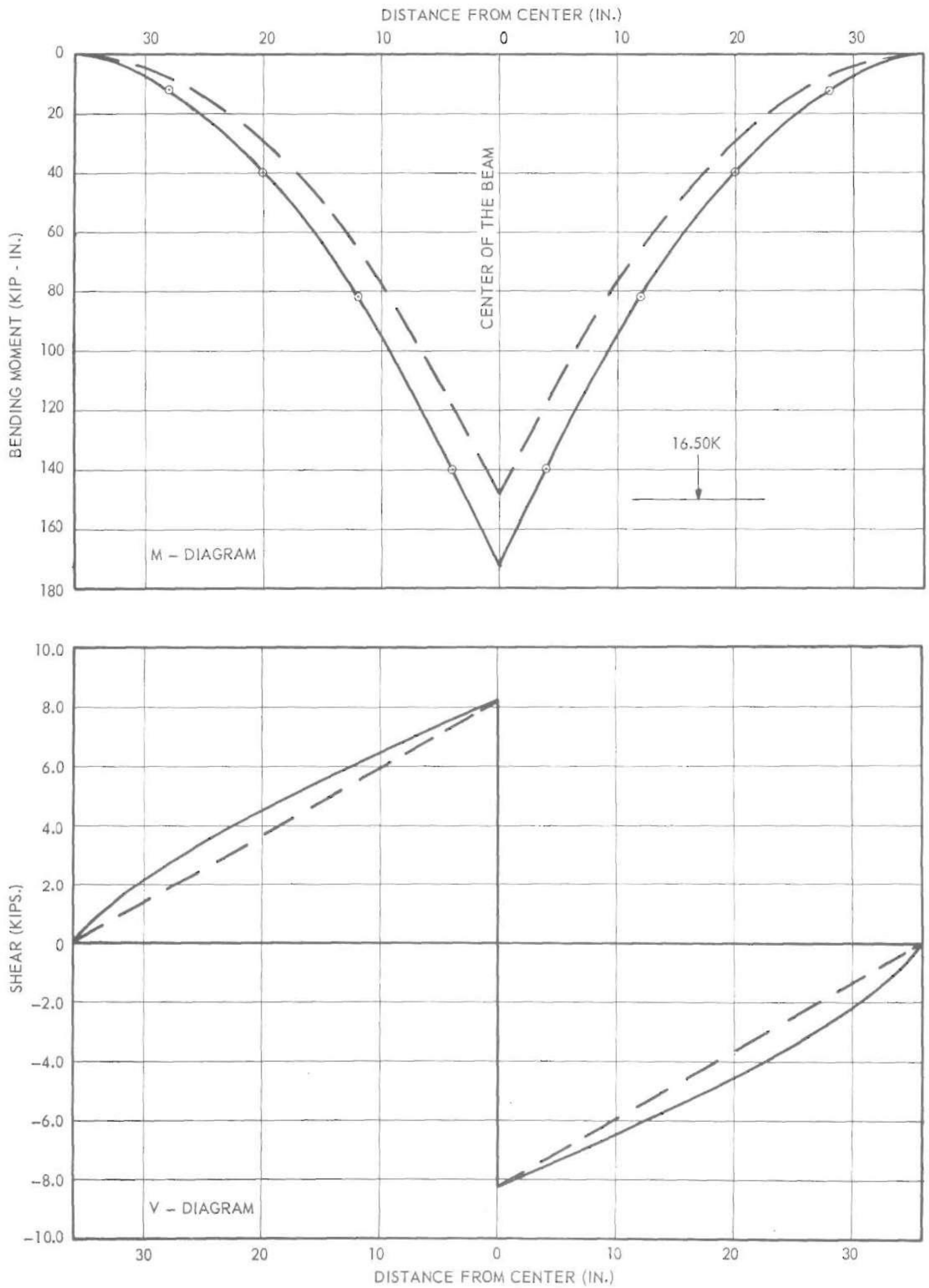


Figure 13. Values of Bending Moment and Shear for $\lambda L = 0.98$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

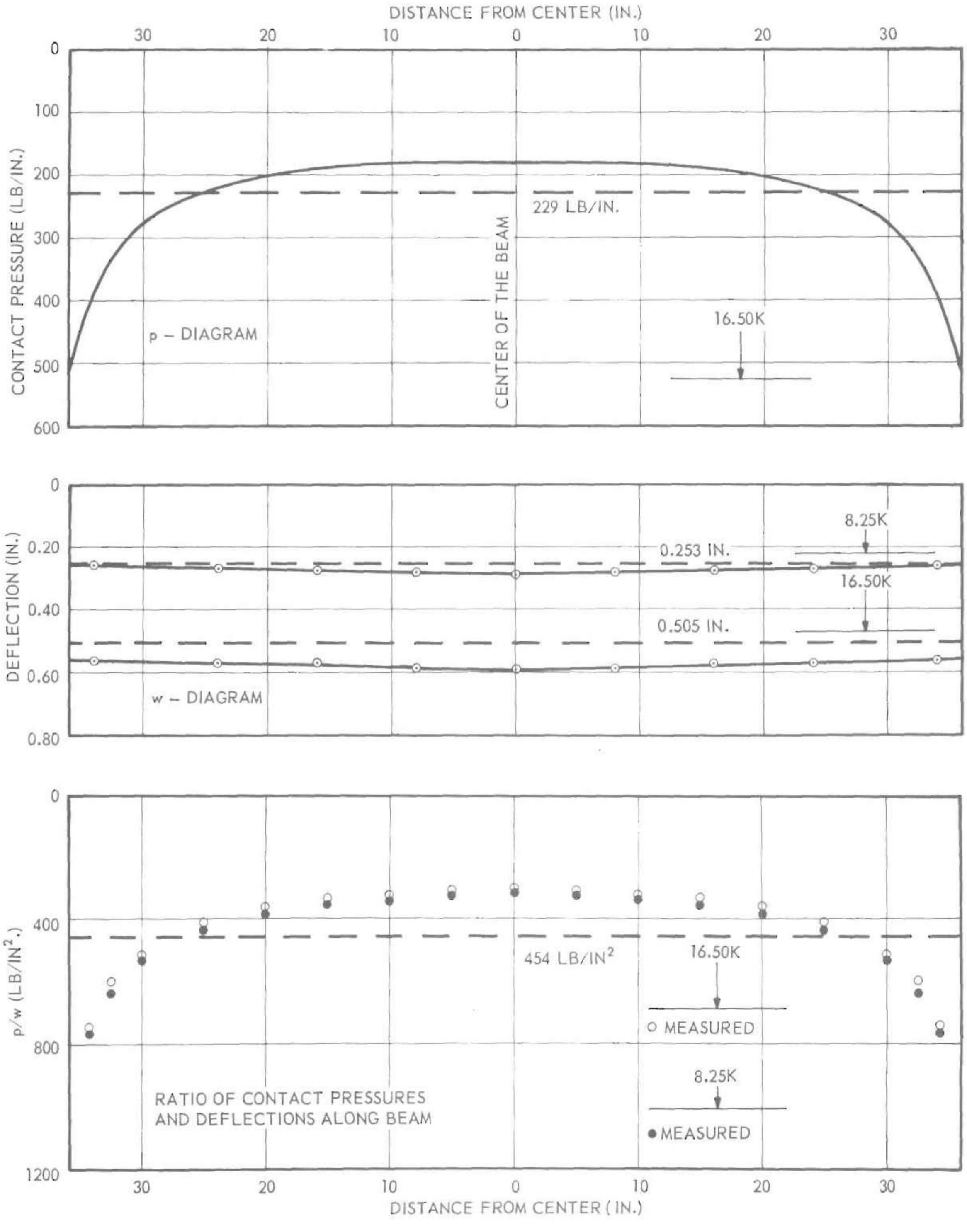


Figure 14. Values of Contact Pressures, Deflections and K for $\lambda L = 0.98$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

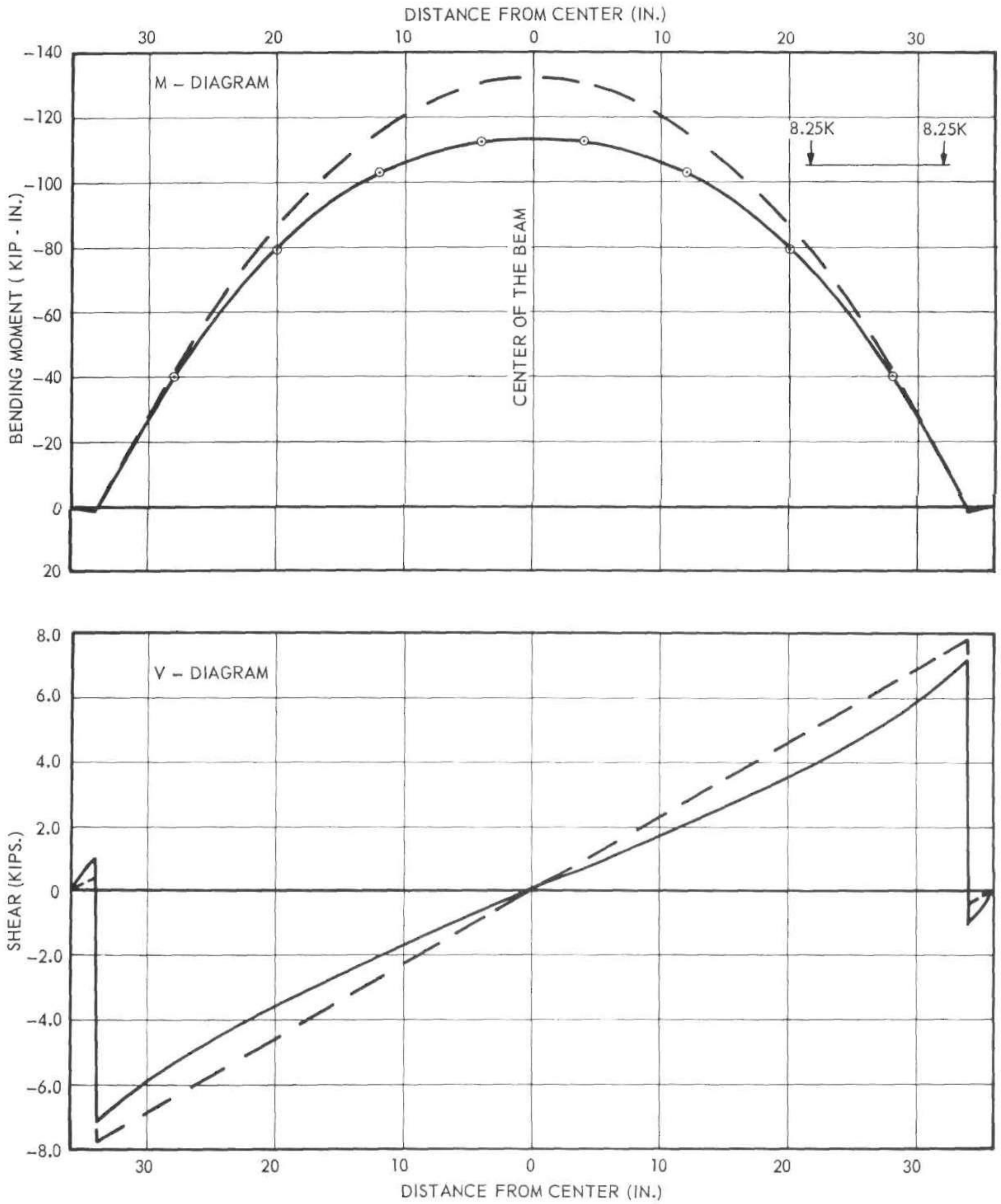


Figure 15. Values of Bending Moment and Shear for $\lambda L = 0.98$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

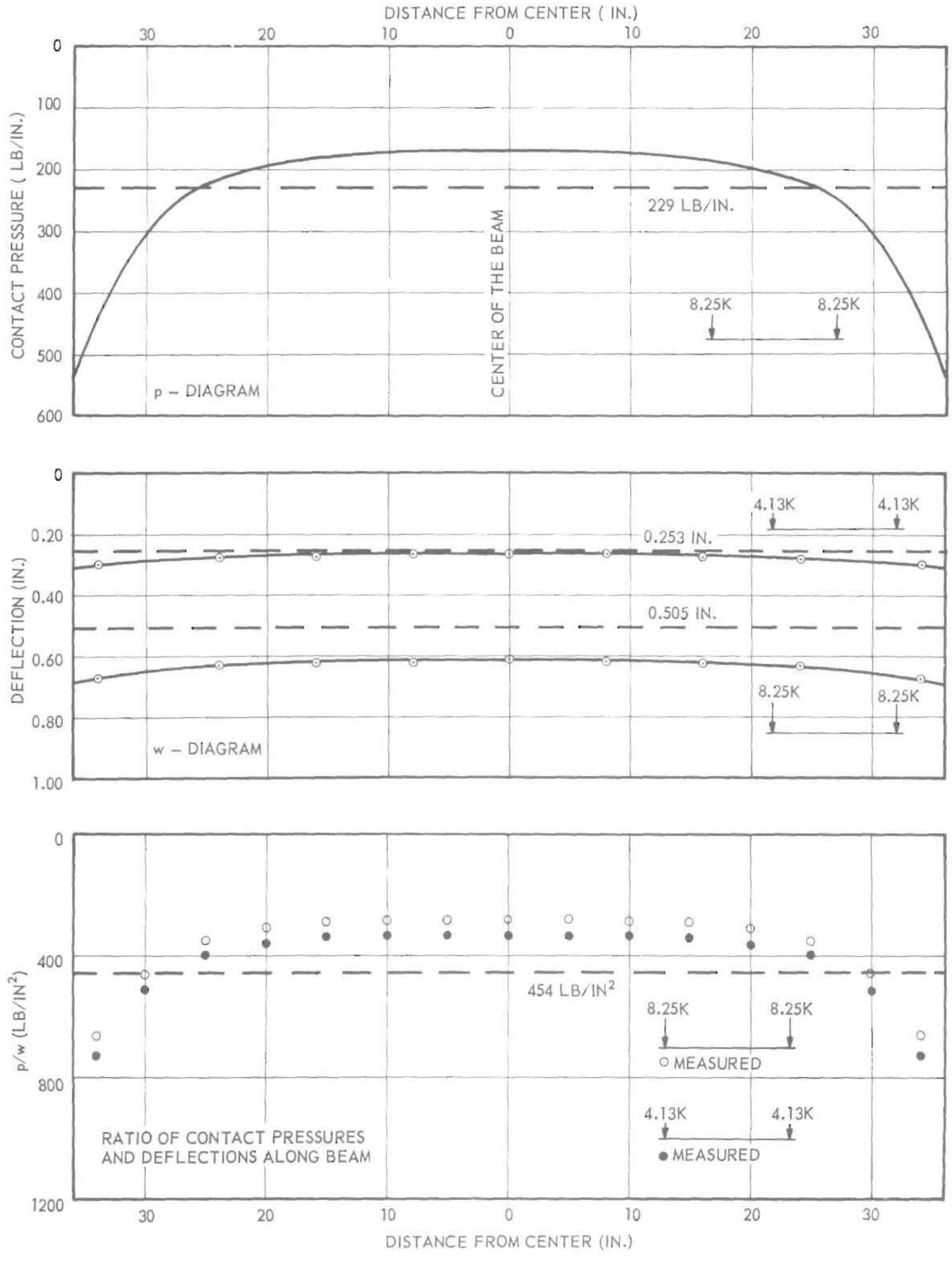


Figure 16. Values of Contact Pressures, Deflections, and K for $\lambda L = 0.98$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

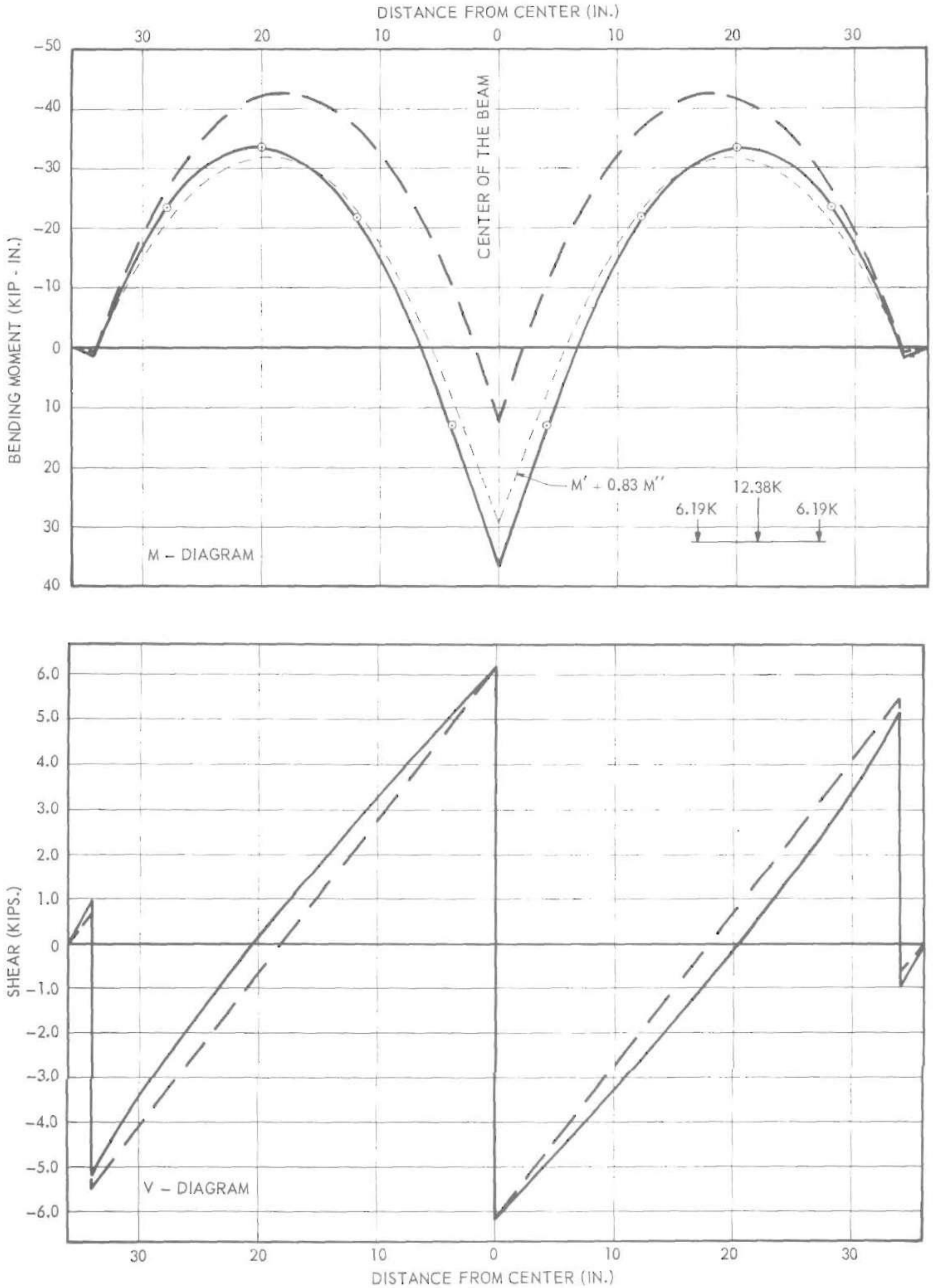


Figure 17. Values of Bending Moment and Shear for $\lambda L = 0.98$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

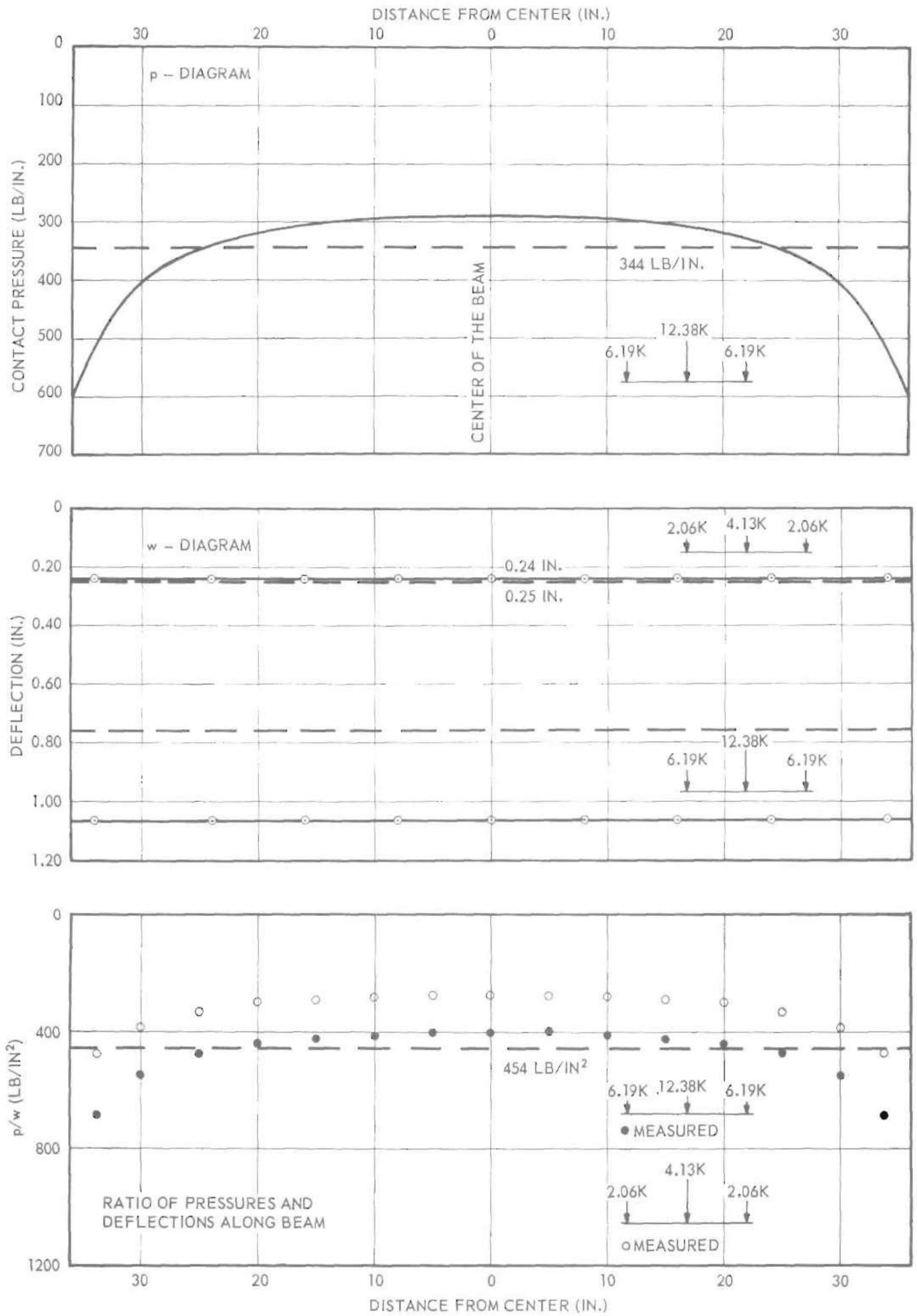


Figure 18. Values of Contact Pressures, Deflections, and K for $\lambda L = 0.98$ as Obtained from Test Results (Full Lines). Dotted lines show corresponding computed values.

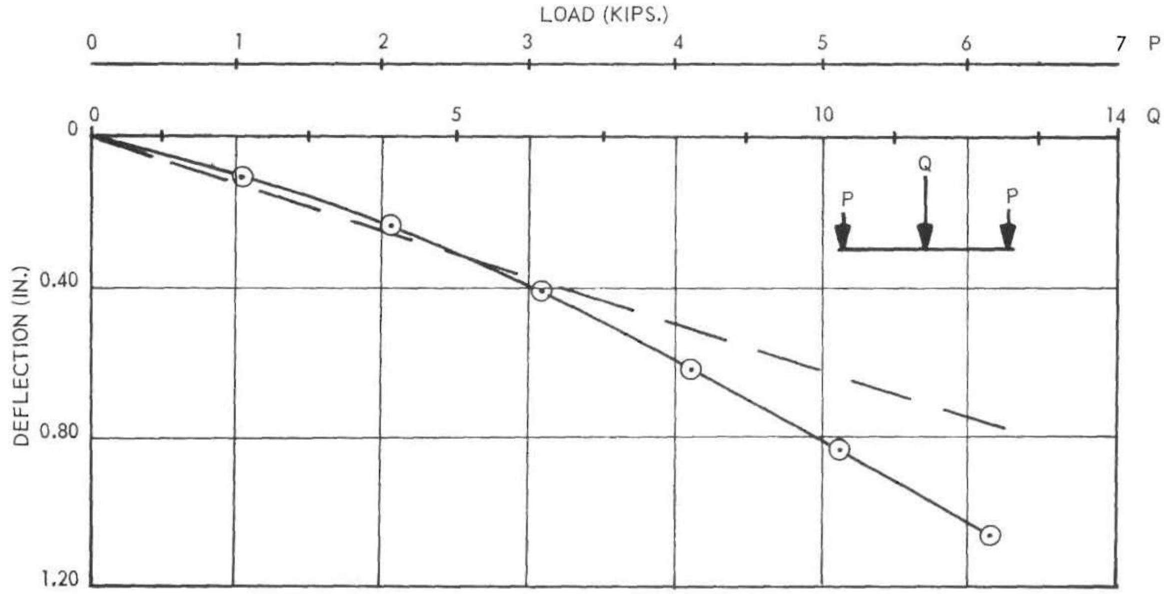


Figure 19. Typical Load vs. Beam Deflection for $\lambda L = 0.98$.

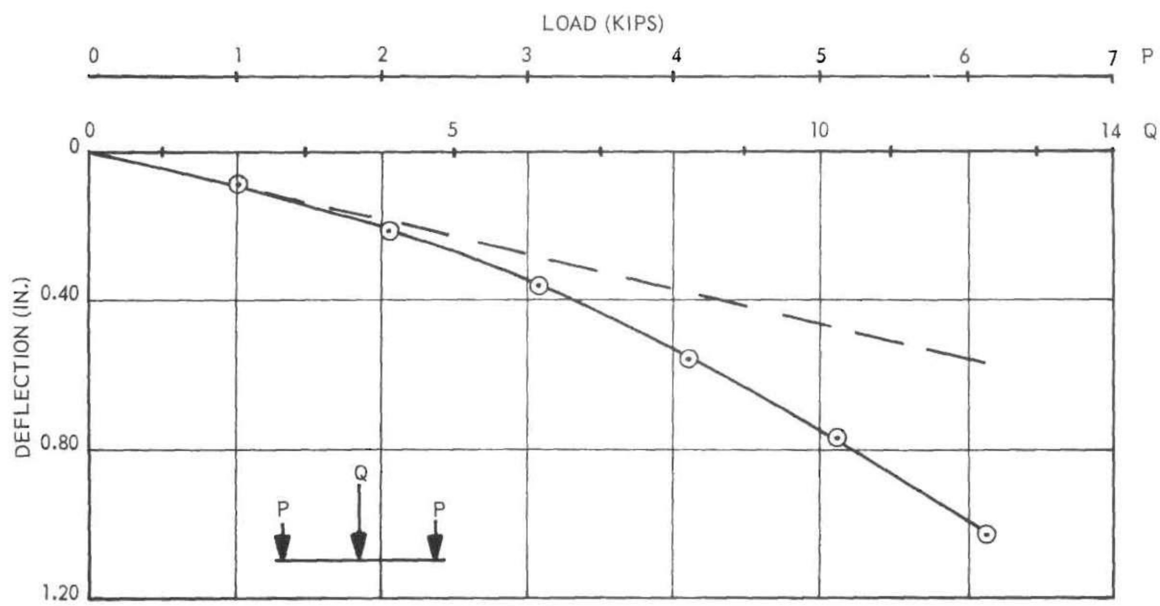


Figure 20. Typical Load vs. Beam Deflection for $\lambda L = 2.40$.

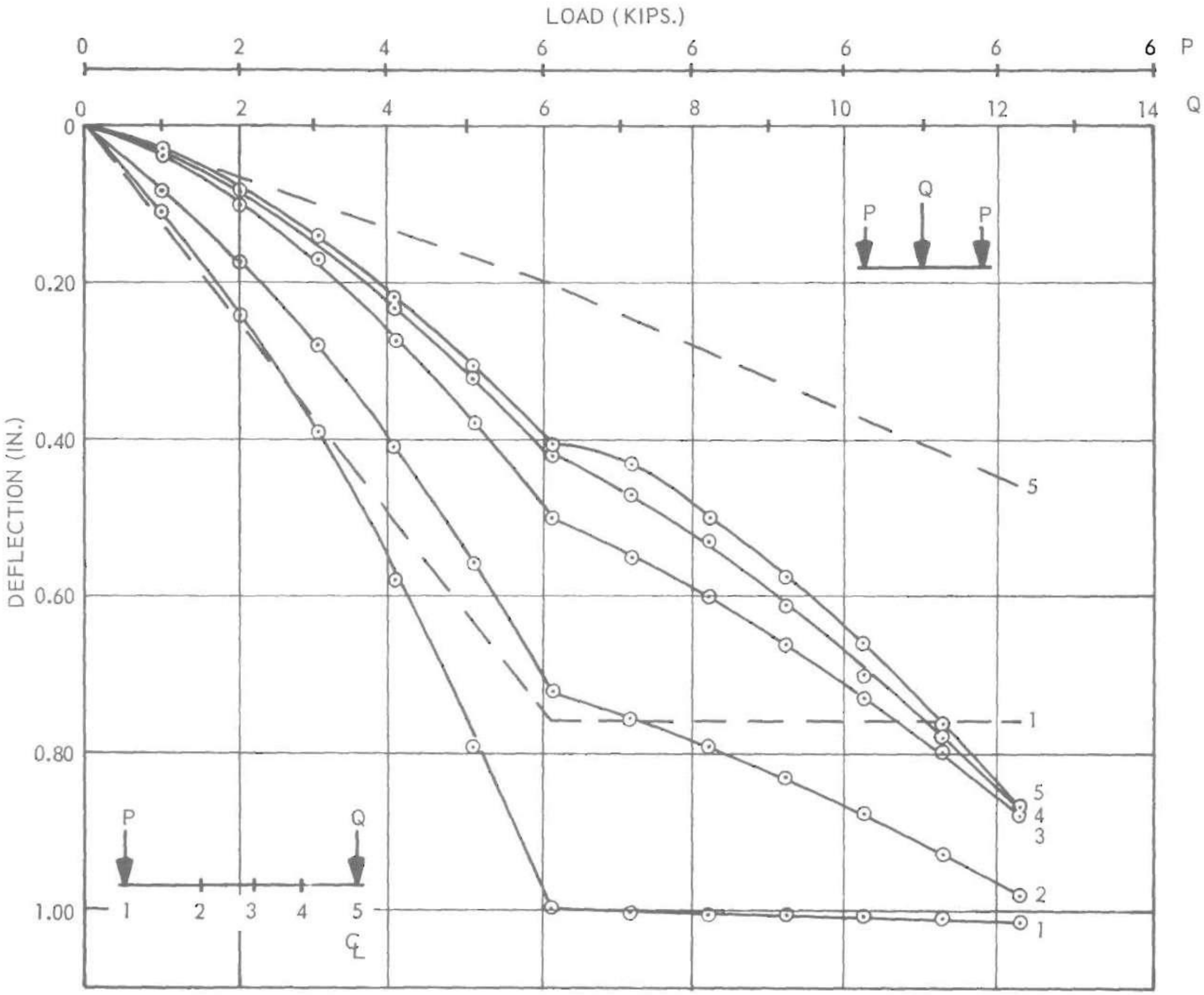


Figure 21. Load vs. Beam Deflection for $\lambda L = 3.91$.

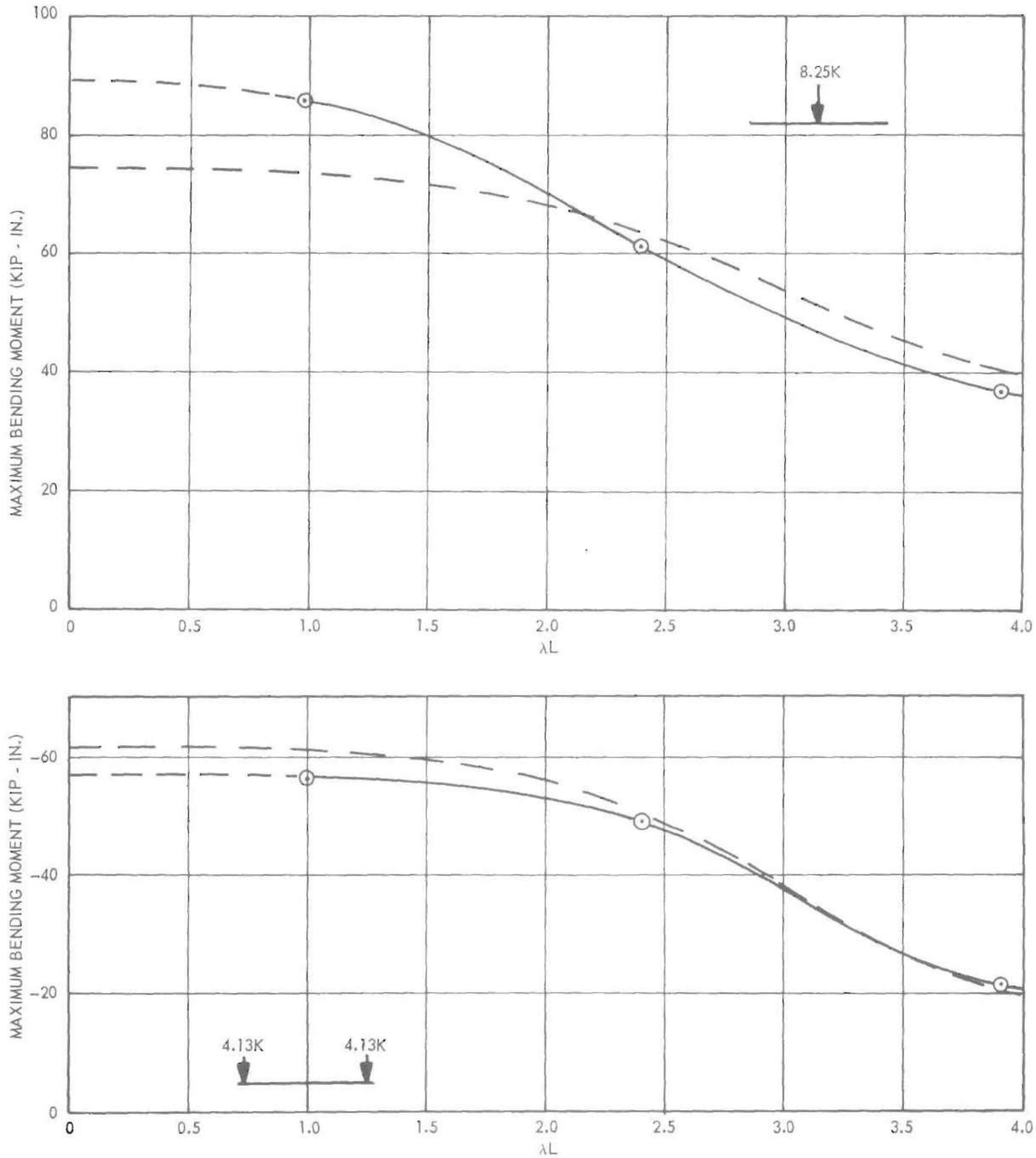


Figure 22. Values of Maximum Bending Moment for Various Values of λL with Loading as Shown. Dotted lines show corresponding computed values.

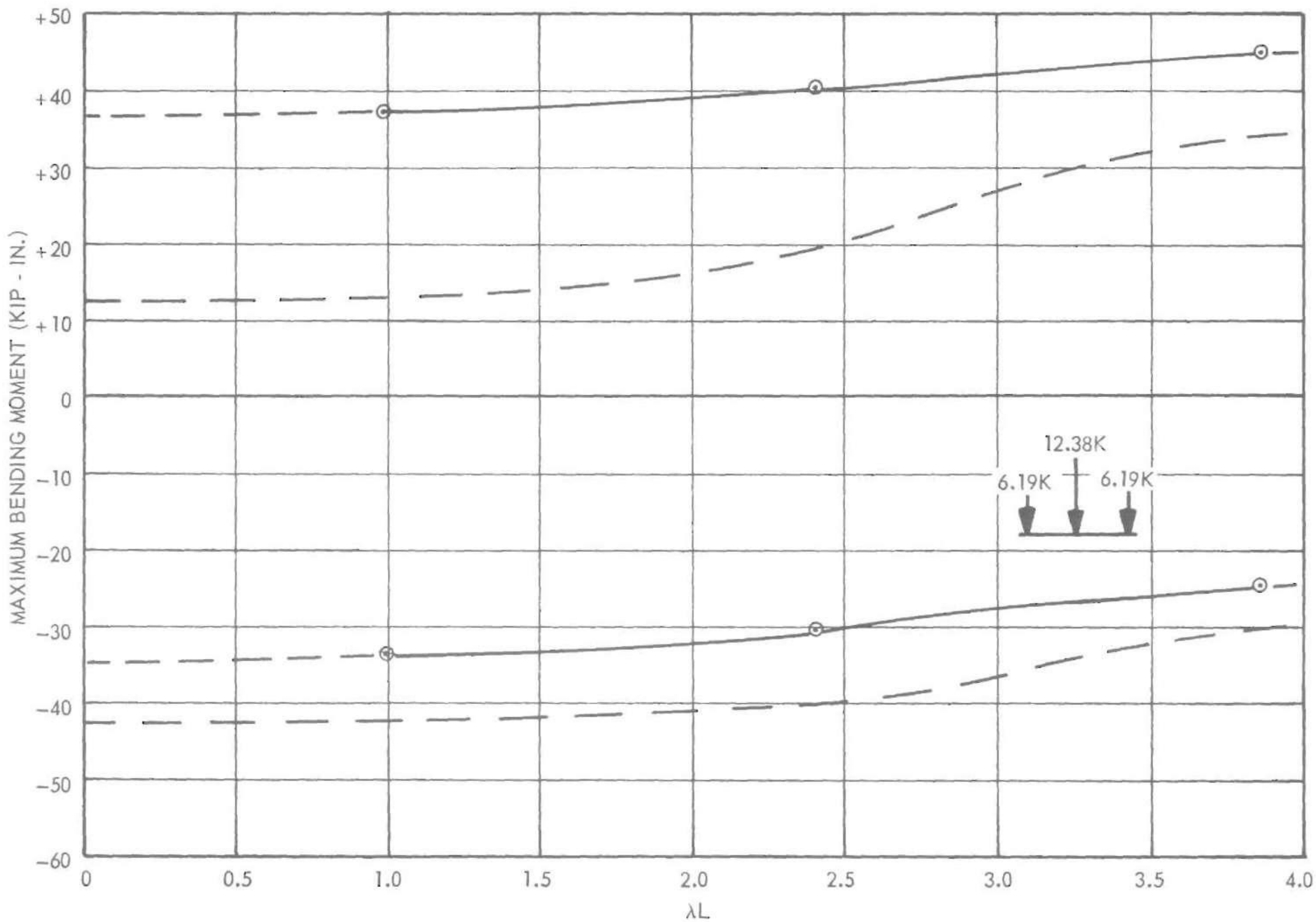


Figure 23. Values of Maximum Negative and Positive Bending Moments for Various Values of λL with Loading as Shown. Dotted lines show corresponding computed values.

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